

$$= \frac{xy(x^2 - 2xy + y^2)}{xy(x^2 - y^2)} = \frac{(x-y)}{(x-y)(x+y)} = \frac{x-y}{x+y}$$

$$c) \frac{a/\frac{1}{2} + \frac{b^2}{a}}{b/\frac{1}{2} + \frac{a^2}{b}} = \frac{\frac{a^2+b^2}{a}}{\frac{b^2+a^2}{b}} = \frac{a^2+b^2}{a} \cdot \frac{b}{b^2+a^2} = \frac{b(a^2+b^2)}{a(b^2+a^2)} = \frac{b}{a}$$

$$b) x^2 - y^2 = 1$$

$$\left(\frac{1}{2}\left(\frac{1}{t} + t\right)\right)^2 - \left(\frac{1}{2}\left(\frac{1}{t} - t\right)\right)^2 = 1, \quad t \neq 0$$

$$\frac{1}{4}\left(\frac{1}{t} + t\right)^2 - \frac{1}{4}\left(\frac{1}{t} - t\right)^2 = 1$$

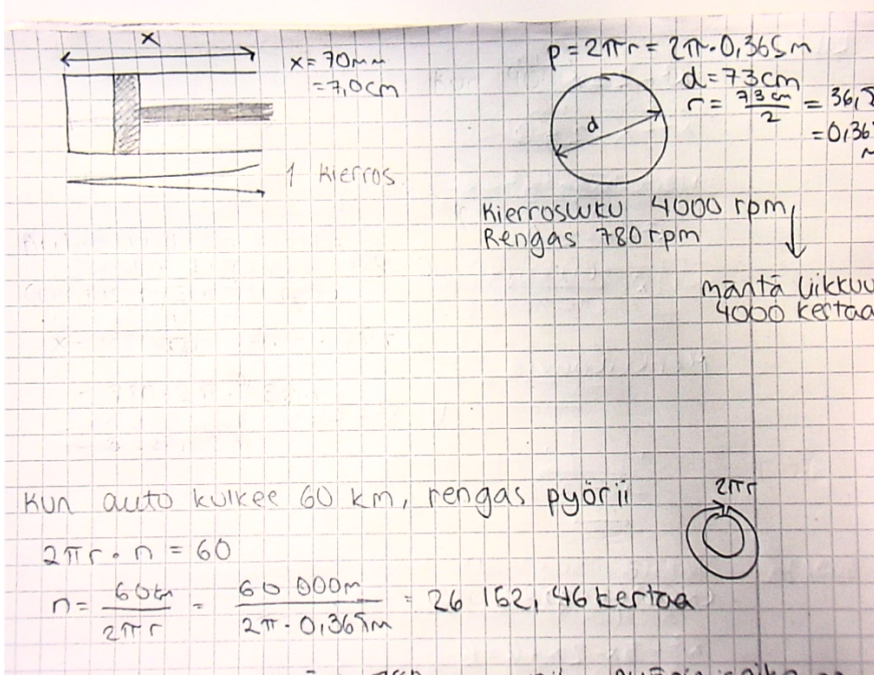
$$\frac{1}{4}\left(\frac{1}{t^2} + 2 \cdot \frac{1}{t} \cdot t + t^2\right) - \frac{1}{4}\left(\frac{1}{t^2} - 2 \cdot \frac{1}{t} \cdot t + t^2\right) = 1$$

$$\frac{1}{4t^2} + \frac{1}{2} + \frac{t^2}{4} - \left(\frac{1}{4t^2} - \frac{1}{2} + \frac{t^2}{4}\right) = 1$$

$$\frac{1}{4t^2} + \frac{1}{2} + \frac{t^2}{4} - \frac{1}{4t^2} + \frac{1}{2} - \frac{t^2}{4} = 1$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$1 = 1 \quad \square$$



$x = 70 \text{ mm} = 7,0 \text{ cm}$
 1 kierros

$p = 2\pi r = 2\pi \cdot 0,365 \text{ m}$
 $d = 73 \text{ cm}$
 $r = \frac{73 \text{ cm}}{2} = 36,5 \text{ cm} = 0,365 \text{ m}$

Kierrosnopeus 4000 rpm,
 Rengas 780 rpm

mäntä liikkuu
 4000 kertaa

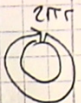
Kun auto kulkee 60 km, rengas pyörii

$2\pi r \cdot n = 60$
 $n = \frac{60 \text{ km}}{2\pi r} = \frac{60 \cdot 1000 \text{ m}}{2\pi \cdot 0,365 \text{ m}} = 26162,46 \text{ kertaa}$

Kunkin renkaas pyörii 780 rpm, niin pyörimisaika on

Kun auto kulkee 60 km, rengas pyörii

$$2\pi r \cdot n = 60$$

$$n = \frac{60 \text{ km}}{2\pi r} = \frac{60 \cdot 1000 \text{ m}}{2\pi \cdot 0,365 \text{ m}} = 26 \, 162,46 \text{ kertaa}$$


Koska rengas pyörii 780 rpm, niin pyörimisaika on

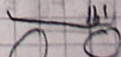
$$\frac{26 \, 162,46 \text{ r}}{780 \text{ r/min}} = 33,5416 \text{ min}$$

Mäntä liikkuu sylinterissä 33,5416 minuutin aikana

$$4000 \text{ r/rev} \cdot 33,5416 \text{ min} = 134 \, 166,4 \text{ kertaa}$$

Jokainen edes-taas liike on 2x, joten männän
kulkema matka sylinterissä

$$s = 2x \cdot 134 \, 166,4 = 2 \cdot 7,0 \text{ cm} \cdot 134 \, 166,4 = 1 \, 878 \, 330 \text{ cm}$$

$$\approx 18,8 \text{ km}$$


$$\sqrt[3]{a} (\sqrt[3]{a^2} - \sqrt[3]{a^5}) = \sqrt[3]{a^{2+1}} - \sqrt[3]{a^{5+3}} = a - a^2$$

5.

Jos $x + y + z = 0$ ja $x^2 + y^2 + z^2 = 1$, niin $xy + yz + zx = -\frac{1}{2}$

niin $(x + y + z)^2 = 0$

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = 0$$

$$(x + y + z) \cdot (x + y + z) = 0$$

Jos $x^2 + y^2 + z^2 = 1$, niin $1 + 2xy + 2yz + 2xz = 0$

$$2xy + 2yz + 2xz = -1$$

$$xy + yz + xz = -\frac{1}{2} \quad \square$$

$$2(xy + yz + xz) = -1 \quad | :2$$

Sokeri	Sokeripitoisuus	Määrä
mehu	0,11	$a+b$
prinsippi	0,14	a
omaa	0,07	b

$$\frac{0,14a + 0,07b}{a+b} = 0,11 \quad | \cdot (a+b), a, b \neq 0$$

$$0,14a + 0,07b = 0,11a + 0,11b$$

$$0,03a = 0,04b \quad | \cdot 100$$

$$3a = 4b \quad | : b$$

$$\frac{3a}{b} = 4 \quad | : 3$$

$$\frac{a}{b} = \frac{4}{3}$$

V: 4 osaa prinsippiä ja 3 osaa omaa