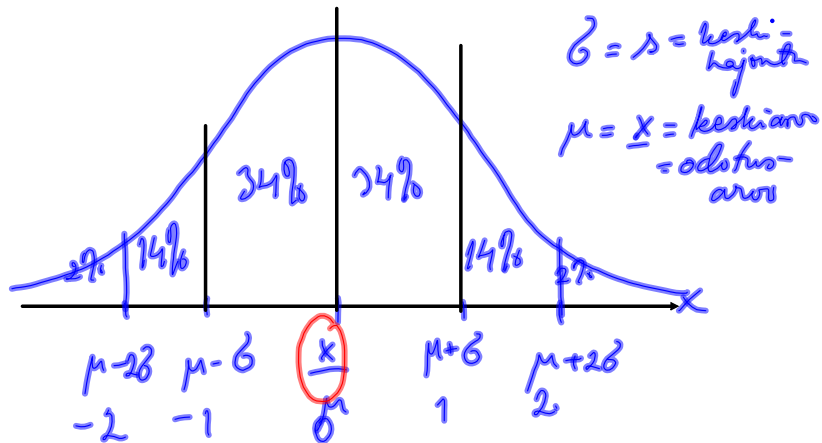


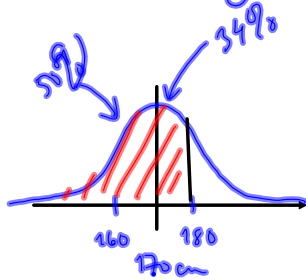
NORMAALIJAKAUMA / SIP



esim Koulun opiskelijat keskipituus 170 cm ja keskihajonta 10 cm.

- a) enintään 180 cm pitkiä
- b) pituudeltaan 150 - 160 cm pitkiä

Ratk. $\mu = 170$ cm keskiarvo
 $\sigma = 10$ cm keskihajonta



$$180 \text{ cm} - 170 \text{ cm} = 10 \text{ cm} = \sigma$$

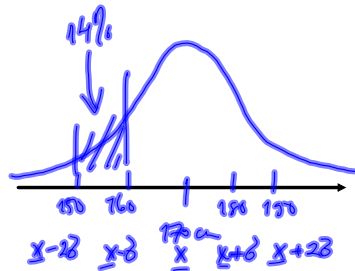
$$\mu + \sigma = 170 + 10 = 180$$

$$P(X \leq 180) = 0,50 + 0,34 = 0,84$$

b) opiskelija 150 cm

$$170 \text{ cm} - 150 \text{ cm} = 20 \text{ cm}$$

$$= 2 \cdot 10 \text{ cm}$$



$$P(150 \leq X \leq 160) = \underline{\underline{14\%}}$$

$$\underline{x} \sim N(\mu, \sigma)$$

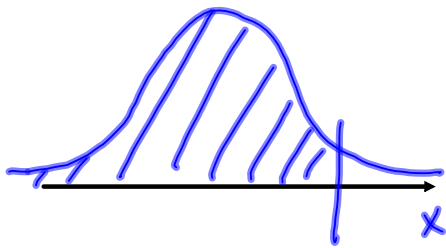
$$\underline{x} \sim N(0, 1)$$

$$\underline{z} = \frac{x - \bar{x}}{s}$$

$$\underline{z} = \frac{\underline{x} - \mu}{\sigma}$$

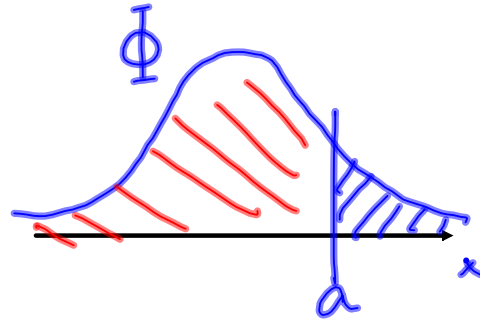
Suoritetaan
normeeraus

kaikissa
 $a > 0$

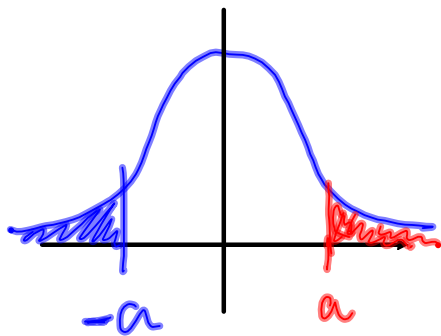


$$P(\underline{x} \leq a) = \Phi(a)$$

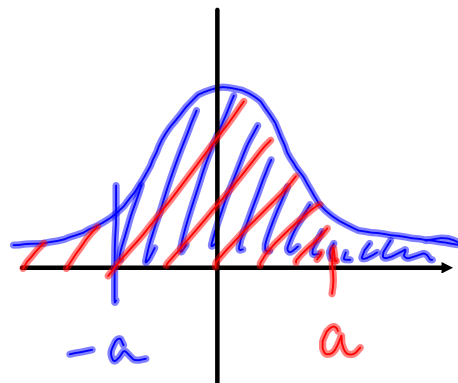
Kertymzfunktio Φ iso ti



$$P(\underline{x} \geq a) = 1 - \Phi(a)$$

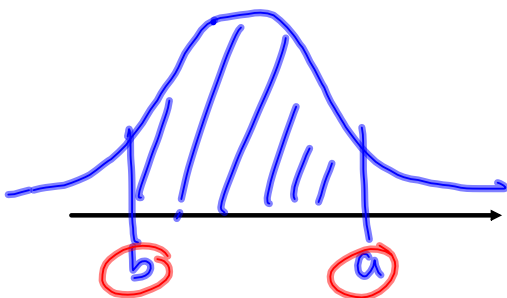


$$P(\underline{x} \leq -a) = P(\underline{x} \geq a) \\ = 1 - \Phi(a)$$



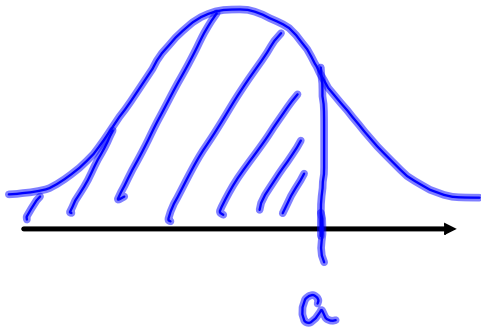
$$P(\underline{x} \geq -a) = P(\underline{x} \leq a) \\ = \Phi(a)$$

SYMMETRISYYS

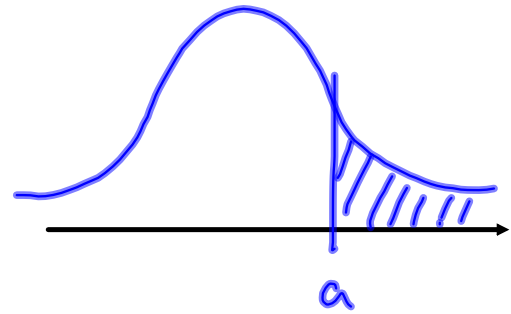


$$P(b \leq \underline{x} \leq a) \\ = P(\underline{x} \leq a) - P(\underline{x} \leq b) \\ = \Phi(a) - \Phi(b)$$

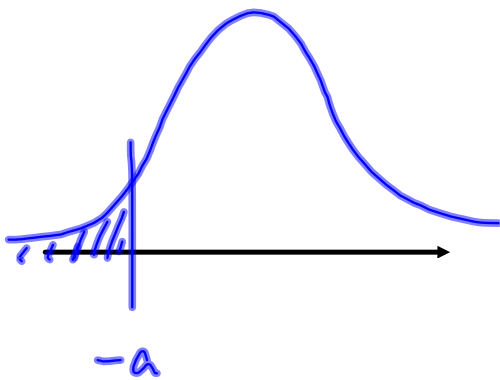
Keretymérfunkció Φ is φ



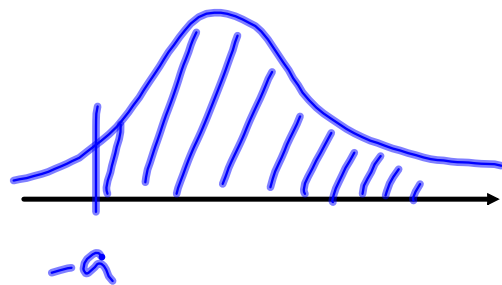
$$P(\underline{x} \leq a) = \Phi(a)$$



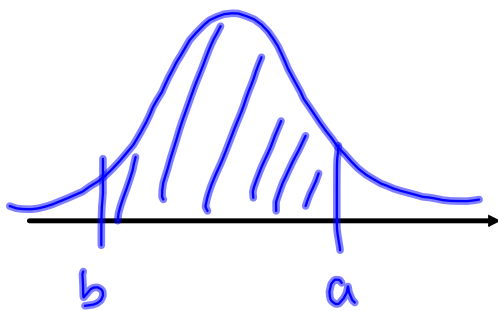
$$P(\underline{x} \geq a) = 1 - \Phi(a)$$



$$\begin{aligned} P(\underline{x} \leq -a) &= \Phi(-a) \\ &= 1 - \Phi(a) \end{aligned}$$



$$\begin{aligned} P(\underline{x} \geq -a) \\ &= P(\underline{x} \leq a) = \Phi(a) \end{aligned}$$



$$\begin{aligned} P(b \leq \underline{x} \leq a) \\ &= \Phi(a) - \Phi(b) \end{aligned}$$

esim 2 Kaupungin asukkaiden (12 vuorokautta) pituus noudattaa normaalijakaumaa $N(162, 35)$ ja kaupungin olevan varuskunnan pituus jakaumaa $N(174, 10)$.

Kuinka monta prosenttia

a) asukkaita on keskipituista sotilasta pidempää?

b) sotilasta on keskipituista asukasta lyhyempää?

Ratk. A: $\underline{x} \sim N(162, 35)$

S: $\underline{y} \sim N(174, 10)$

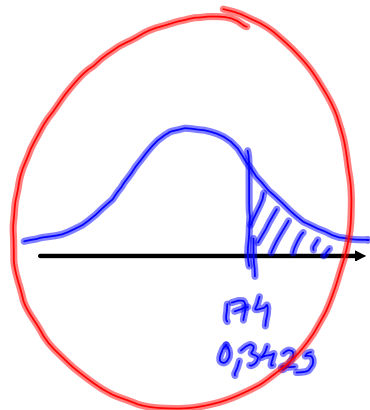
$$a) \underline{z} = \frac{\underline{x} - \mu}{\sigma} = \frac{174 - 162}{35}$$

$$P(\underline{x} \geq 174) = 1 - P(\underline{x} < 174)$$

$$= 1 - \Phi(0,3429)$$

$$= 1 - 0,6331$$

$$= 0,3669 \approx \underline{\underline{0,37}}$$



S. 168 - 172

245, 247, 251, 252, 256

$$b) \quad \underline{\underline{z}} = \frac{162 - 174}{10}$$

$$= -\frac{12}{10} = -1,2$$

$$P(y \leq 162)$$

$$= P(y \geq 162)$$

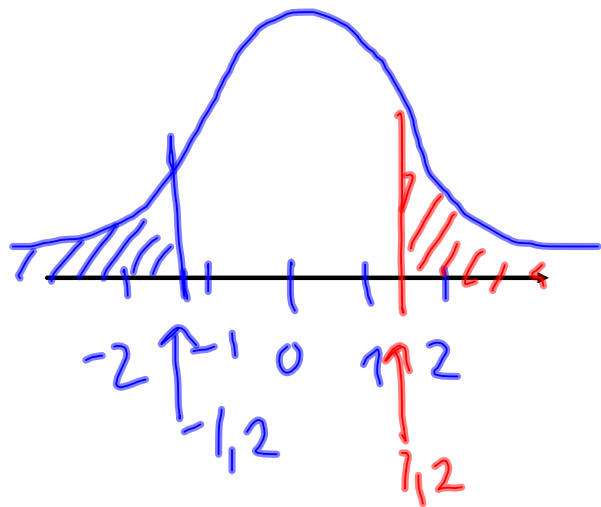
$$= 1 - P(y < 162)$$

$$= 1 - \Phi(1,2)$$

$$= 1 - 0,8849$$

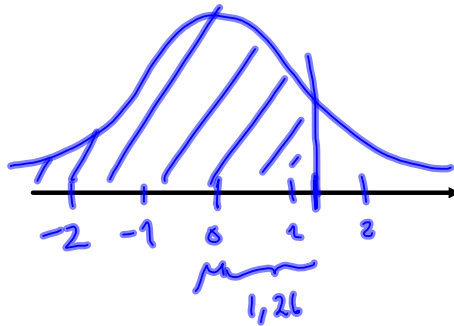
$$= 0,1151$$

$$\approx \underline{\underline{0,12}}$$



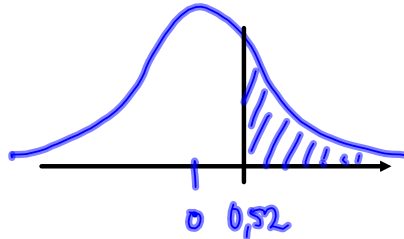
exm a) $P(Z < 1,26)$

$$= \Phi(1,26) = 0,8962$$
$$= \underline{\underline{0,896}}$$



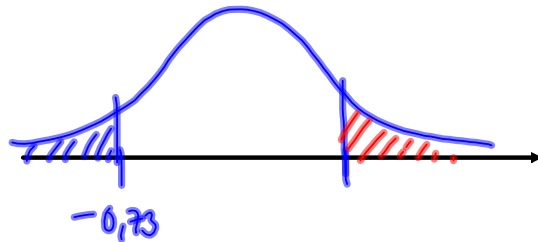
b) $P(Z \geq 0,52)$

$$= 1 - P(Z < 0,52)$$
$$= 1 - \Phi(0,52)$$
$$= 1 - 0,6985 = 0,3015$$
$$= \underline{\underline{0,30}}$$



c) $P(Z < -0,73)$

$$= P(Z > 0,73)$$
$$= 1 - P(Z \leq 0,73)$$
$$= 1 - \Phi(0,73)$$
$$= 1 - 0,7673$$
$$= \underline{\underline{0,23}}$$



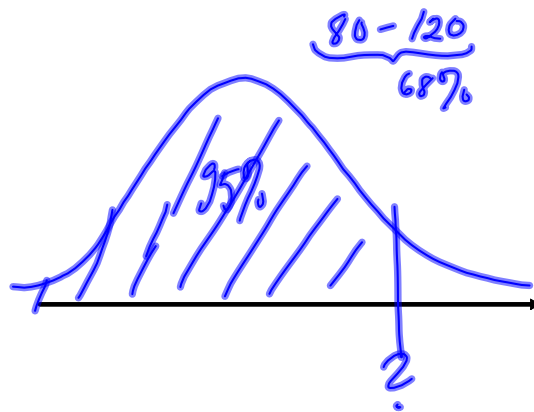
esim Suomalaisen älykkyyssummäärän keskimäärin 100 keskihajonnan ollessa 20. Minkä älykkyyssummäärän alapuolella on 95% suomalaisista?

Ratk. Kysytty älykkyyssummäärä on a
normeeraus:

$$z = \frac{x - \mu}{\sigma} \left(= \frac{x - \bar{x}}{s} \right) \quad \left| \begin{array}{l} x = a \\ \mu = 100 \\ \sigma = 20 \end{array} \right.$$

$$z = \frac{a - 100}{20}$$

taulukosta nähdään,
että tod.näk. 0,95
vastaa z :n arvo 1,65



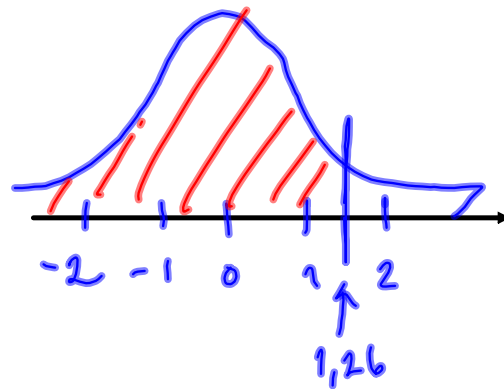
$$\frac{a - 100}{20} = 1,65 \quad | \cdot 20$$

⋮

$$a = 133$$

V: Älykkyyssummäärän 133 alapuolella on 95% suomalaisista.

errin a) $P(Z < 1,26)$
 $= \Phi(1,26)$
 $= \underline{\underline{0,8962}}$



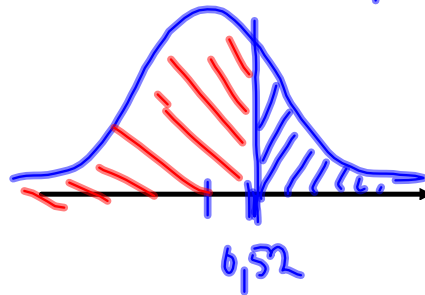
b) $P(Z \geq 0,52)$

$= 1 - P(Z < 0,52)$

$= 1 - \Phi(0,52)$

$= 1 - 0,6985$

$= 0,3015 \approx 0,30$



c) $P(Z < -0,73)$



$= 1 - 0,7673$

$= \underline{\underline{0,23}}$