DERIVOINTI – JA INTEGROINTIKERTAUS
RATKAISUT

1. **a) D x^{10} =** $10x^9$

b) D ($6x^3 - x - e^x - e^2$)= $18x^2 - 1 - e^x$

c) D cos 2x= $-2\sin 2x$

d) D $e^{x^2} = e^{x^2} \cdot D(x^2) = 2xe^{x^2}$

2. **a) D($x^3 * e^x$) =** $3x^2 \cdot e^x + x^3 \cdot e^x = e^x x^2 (x+3)$

b) D ln x= $\frac{1}{x}$**, x > 0**

c) D ln (3x + 1)= $\frac{1}{3x+1} D(3x+1) = \frac{3}{3x+1}$, $x > -\frac{1}{3}$

d) D($x^2 - 1)^3(1 - x)$

$= 3(x^2-1)^2 \cdot 2x \cdot (1-x) + (x^2-1)^3(-1)$

$= (x^2-1)^2(6x - 6x^2 - x^2 + 1) = (x^2-1)^2(-7x^2 + 6x + 1)$

3. **a)** $D\frac{x^2+x}{2x^2+1} = \frac{(2x+1)(2x^2+1) - (x^2+x)\cdot 4x}{(2x^2+1)^2} = \frac{-2x^2+2x+1}{4x^4+4x^2+1}$

b) $D\sqrt[3]{x^2} = Dx^{\frac{2}{3}} = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3}\cdot\frac{1}{x^{\frac{1}{3}}} = \frac{2}{3\sqrt[3]{x}} = \frac{2(\sqrt[3]{x})^2}{3x}$, $x \neq 0$

c)

$D(\sqrt{-x} - \sqrt{1+2x}) = D((-x)^{\frac{1}{2}} - (1+2x)^{\frac{1}{2}}) = \frac{1}{2}(-x)^{-\frac{1}{2}} \cdot (-1) - \frac{1}{2}(1+2x)^{-\frac{1}{2}} \cdot 2$

$= -\frac{1}{2\sqrt{-x}} - \frac{1}{\sqrt{1+2x}} = \frac{\sqrt{-x}}{2x} - \frac{\sqrt{1+2x}}{1+2x}$, $-\frac{1}{2} \le x \le 0$

d) $D\sqrt{1-x^2} = D(1-x^2)^{\frac{1}{2}} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = -\frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{1-x^2}}{1-x^2}$, $-1 < x < 1$

4. **a)** $D\sin\frac{x}{3} = \frac{1}{3}\cos\frac{x}{3}$

b) $D\frac{2}{3}\cos 7x = \frac{2}{3}(-\sin 7x)\cdot 7 = -\frac{14}{3}\sin 7x$

c) $D\sin^2(1-\pi x) = 2\sin(1-\pi x)\cos(1-\pi x)(-\pi)$

$= -\pi 2\sin(1-\pi x)\cos(1-\pi x) = -\pi\sin 2(1-\pi x) = -\pi\sin(2-2\pi x)$

d) $D\sin(3x-2)^2 = \cos(3x-2)^2 D(3x-2)^2 = (18x-12)\cos(3x-2)^2$

5. **a)** $\int 5x^2 dx = \frac{5}{3}x^3 + C$

b) $\int (t + \cos x)dx = tx + \sin x + C$

c) $\int x \sin t \ dx = \sin t \int x dx = \frac{1}{2}x^2 \sin t + C$

d) $\int x \sin t \ dt = x \int \sin t dt = x(-\cos t) + C = -x \cos t + C$

6. **a)** $\int (7x^3 - \frac{1}{2}x^2 + 3)dx = \frac{7}{4}x^4 - \frac{1}{2} \cdot \frac{1}{3}x^3 + 3x + C = \frac{7}{4}x^4 - \frac{1}{6}x^3 + 3x + C$

b) $\int \sqrt[5]{x} dx = \int x^{\frac{1}{5}} dx = \frac{5}{6}x^{\frac{6}{5}} + C = \frac{5}{6}x\sqrt[5]{x} + C$

c) $\int \frac{4}{x^4} dx = 4 \int x^{-4} dx = 4 \cdot (-\frac{1}{3}x^{-3}) + C = -\frac{4}{3x^3} + C, x \neq 0$

d) $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C, x \neq 0$

7. **a)** $\int (5x-2)^6 dx = \frac{1}{5} \int 5 \cdot (5x-2)^6 dx = \frac{1}{5} \cdot \frac{1}{7}(5x-2)^7 + C = \frac{1}{35}(5x-2)^7 + C$

b) $\int \frac{1}{1-x} dx = -\int \frac{-1}{1-x} dx = -\ln|1-x| + C, x \neq 1$

c) $\int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln|\ln x| + C, x > 0$ **, x ≠ 1**

d) $\int \frac{\ln x}{x} dx = \int \ln x D(\ln x) dx = \frac{1}{2}(\ln x)^2 + C, x > 0$ **, x ≠ 1**

8. **a)**

$\int 2^{-3x} dx = (-\frac{1}{3}) \int (-3) \cdot 2^{-3x} dx = (-\frac{1}{3}) \cdot \frac{2^{-3x}}{\ln 2} + C = -\frac{1}{2^{3x} \cdot 3 \ln 2} = -\frac{1}{2^{3x} \ln 8}$, $(a > 0, a \neq 1)$

b) $\int \cos 2x dx = \frac{1}{2} \int 2 \cos 2x dx = \frac{1}{2} \sin 2x + C$

c) $\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x + C$

d) $\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$