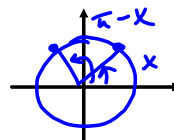


70a) ~~1 typpi~~ $\sin 2x = \sin x$



$$2x = x + n2\pi$$

$$\text{tai } 2x = \pi - x + n2\pi$$

$$2x - x = n2\pi$$

$$\text{tai } 2x + x = \pi + n2\pi$$

$$x = n2\pi$$

$$\text{tai } 3x = \pi + n2\pi \quad | :3$$

$$\checkmark: x = n2\pi$$

$$\text{tai } x = \frac{\pi}{3} + n \cdot \frac{2\pi}{3}$$

$$\begin{aligned} x=0 & \quad n=0 \\ x=2\pi & \quad n=1 \\ x=4\pi & \quad n=2 \end{aligned}$$

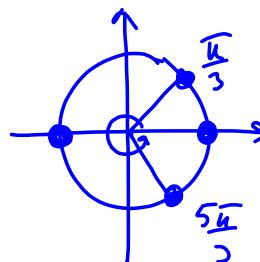
$$x = \frac{\pi}{3} + n \cdot \frac{2\pi}{3}$$

$$x = \frac{\pi}{3} \quad n=0$$

$$x = \pi \quad n=1$$

$$x = \frac{\pi}{3} + 2 \cdot \frac{2\pi}{3} \quad n=2$$

$$= \frac{\pi}{3} + \frac{4\pi}{3} = \frac{5\pi}{3}$$



b) $\cos 3x = \cos x$

$$3x = \pm x + n \cdot 2\pi$$

$$3x = x + n2\pi \quad \text{tai} \quad 3x = -x + n2\pi$$

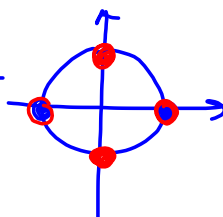
$$3x - x = n2\pi \quad \text{tai} \quad 3x + x = n2\pi$$

$$2x = n2\pi \quad | :2 \quad \text{tai} \quad 4x = n2\pi \quad | :4$$

$$x = n \cdot \pi$$

$$\text{tai} \quad x = n \cdot \frac{\pi}{2}$$

n	x
0	0
1	π
2	2π
3	3π



n	x
0	0
1	$\frac{\pi}{2}$
2	π
3	$\frac{3\pi}{2}$

$$\checkmark: x = n \cdot \frac{\pi}{2}$$

$$\sin \underline{2x} = \sin \underline{x} \quad \underline{\underline{\text{II}}}$$

$$\sin 2x = 2 \sin x \cos x$$

TRIG. FUNKTIOIDEN MUUNNOSKAAVAT

$$\textcircled{1} \quad \sin 2\alpha = 2 \sin \alpha \cos \alpha \quad \nabla$$

$$\textcircled{2} \quad \begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha \end{aligned}$$

$$\textcircled{3} \quad \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

+ 3 muuta kaavaa ∇

esim. $\textcircled{4} \quad \sin(\alpha \pm \beta) = \sin \alpha \cdot \cos \beta \pm \cos \alpha \cdot \sin \beta$
 $\textcircled{5}$
 $\textcircled{6}$

70a) $\sin 2x = \sin x$ \parallel tyhj

$$\begin{aligned} 2 \sin x \cos x - \sin x &= 0 \\ \sin x (2 \cos x - 1) &= 0 \end{aligned}$$

TULON NOLLASÄÄNTÖ

$$\sin x = 0 \quad \text{tai} \quad 2 \cos x - 1 = 0$$

$$\begin{aligned} x &= n2\pi \quad \text{tai} \quad 2 \cos x = 1 \\ \text{tai} \quad \cos x &= \frac{1}{2} \end{aligned}$$

$$x = \bar{n} + n2\pi$$

$$x = \pm \frac{\pi}{3} + n2\pi$$

$$\nabla: \quad n\bar{n} \quad \text{tai} \quad x = \frac{\pi}{3} + n2\pi, \quad n \in \mathbb{Z}$$

$$\begin{aligned} \frac{2 \cancel{\sin x} \cos x}{\cancel{\sin x}} \\ &= 2 \cos x \\ - \frac{\cancel{\sin x}}{\cancel{\sin x}} &= -1 \end{aligned}$$

esim 2 Määritä lausekkeen $\sin(2x + \frac{\pi}{6})$

tarkeka arvo, kun $\tan x = 2$.

$$\alpha = 2x$$

$$\beta = \frac{\pi}{6}$$

Ratk. $\sin(\alpha + \beta)$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \sin 2x \cdot \cos \frac{\pi}{6} + \cos 2x \cdot \sin \frac{\pi}{6}$$

$$= \sin 2x \cdot \frac{\sqrt{3}}{2} + \cos 2x \cdot \frac{1}{2}$$

$$= \cancel{2} \sin x \cos x \cdot \frac{\sqrt{3}}{\cancel{2}} + (2 \cos^2 x - 1) \cdot \frac{1}{2}$$

$$= \sqrt{3} \cdot \sin x \cos x + \frac{1}{2} \cdot \cancel{2} \cos^2 x - \frac{1}{2}$$

$$= \sqrt{3} \frac{\sin x \cdot \cos^2 x}{\cos x} + \frac{1}{1 + \tan^2 x} - \frac{1}{2}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$

jne. kotona

esim 3 $2 \sin^2 \alpha - 4 \sin \alpha + 1 = 0$

2. asteen yhtälö

S 34-35
cdell. +

84
85 (92
87 89)
88
90

K03/5

$$\lg x + \lg(x+30) = 3$$

$$\lg a + \lg b = \lg a \cdot b$$

$$\lg x \cdot (x+30) = 3$$

$$\log_{10}(x^2 + 30x) = 3$$

$$10^3 = x^2 + 30x$$

$$x^2 + 30x - 1000 = 0$$

⋮
⋮

mj:

$$x > 0 \text{ ja } x+30 > 0$$

$$x > 0 \text{ ja } x > -30$$

$$x > 0$$

vertaa

