

## Trigonometristen funktioiden kuvaajat

Sinikäyrän animointi



käynn alkua

laskimella

oim Piirrä laskimelle

a)  $\sin x$

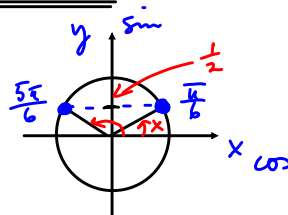
b)  $\sin 2x$

Jos kulma kaksinkertaistuu,  
mitä jaksolle tapahtuu?

Trigonometriset yhtälöt

Yhtälöt  $\sin x = \text{vakio}$   
 $\cos x = \text{vakio}$

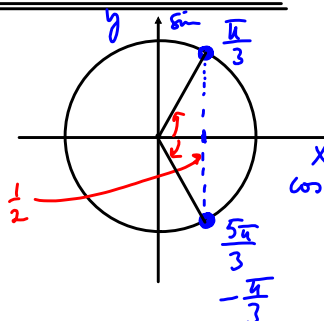
esim 1  $\sin x = \frac{1}{2}$



$x = \frac{\pi}{6} + n \cdot 2\pi$  tai  $x = \pi - \frac{\pi}{6} + n \cdot 2\pi, n \in \mathbb{Z}$

V:  $x = \frac{\pi}{6} + n \cdot 2\pi$  tai  $x = \frac{5\pi}{6} + n \cdot 2\pi$

esim 2  $\cos x = \frac{1}{2}$



$x = \frac{\pi}{3} + n \cdot 2\pi$  tai

$x = \frac{5\pi}{3} + n \cdot 2\pi$

V:  $x = \frac{\pi}{3} + n \cdot 2\pi$  tai  $x = \frac{5\pi}{3} + n \cdot 2\pi$

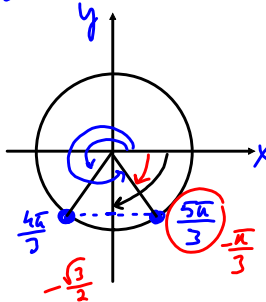
V:  $x = \pm \frac{\pi}{3} + n \cdot 2\pi$

esim 3 Ratkaise yhtälö

$\sin x = \frac{-\sqrt{3}}{2}, -\frac{\pi}{2} < x < 2\pi$

$(-\frac{\sqrt{3}}{2} \approx -0,9)$

$x = \frac{4\pi}{3} + n \cdot 2\pi$  tai  $x = \frac{5\pi}{3} + n \cdot 2\pi$



$\left( \begin{array}{l} \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \\ \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2} \end{array} \right)$

$x = \frac{4\pi}{3} + n \cdot 2\pi$  tai  $x = \frac{5\pi}{3} + n \cdot 2\pi$

n	$\frac{4\pi}{3} + n \cdot 2\pi$	n	$\frac{5\pi}{3} + n \cdot 2\pi$
0	$\frac{4\pi}{3} + 0 \cdot 2\pi = \frac{4\pi}{3}$ käng	0	$\frac{5\pi}{3} + 0 \cdot 2\pi = \frac{5\pi}{3}$ käng
1	$\frac{4\pi}{3} + 2\pi$ ei käng	1	$\frac{5\pi}{3} + 2\pi = \frac{11\pi}{3}$ käng
2	$\frac{4\pi}{3} + 4\pi$ ei käng	2	$\frac{5\pi}{3} + 4\pi$ ei käng
-1	$\frac{4\pi}{3} - 2\pi = \frac{4\pi}{3} - \frac{6\pi}{3} = -\frac{2\pi}{3}$ ei käng	-1	$\frac{5\pi}{3} - 2\pi$ ei käng

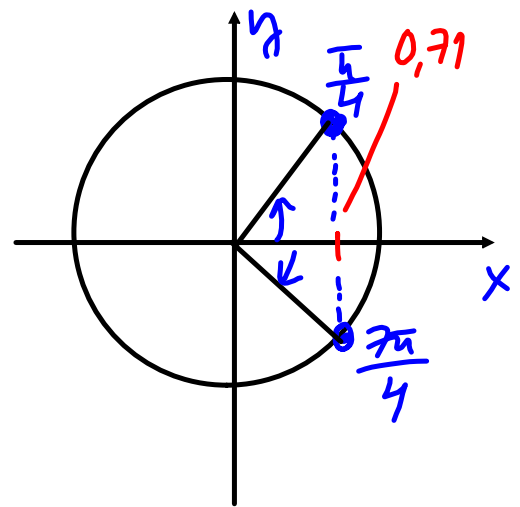
V:  $\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{5\pi}{3}$

$$\cos 2x = \frac{1}{\sqrt{2}} \approx 0,71 \quad | \quad \cos \theta = \frac{1}{\sqrt{2}}$$

$$2x = \frac{\pi}{4} + n2\pi \quad | :2 \quad \text{tai} \quad 2x = \frac{7\pi}{4} + n2\pi \quad | :2$$

$$V: \quad x = \frac{\pi}{8} + n\pi \quad \text{tai} \quad x = \frac{7\pi}{8} + n\pi$$

$$V: \quad \pm \frac{\pi}{8} + n\pi, \quad n \in \mathbb{Z}$$



kt:  
ti kootaan

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59  
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+ piirä  
yhd. ymp.  
kaikeksi

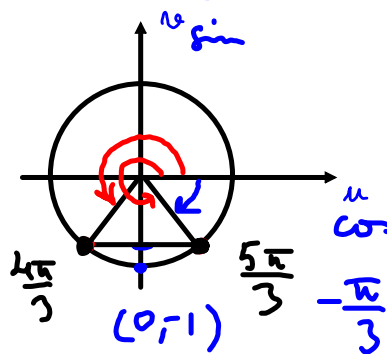
esim Ratkaise yhtälö

$$\sin x = -\frac{\sqrt{3}}{2}, \quad -\frac{\pi}{2} < x < 2\pi.$$

lets.  
s.51

$$-\frac{\sqrt{3}}{2} \approx -0,87$$

- yks. ymp.  
- MAOL



$$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$x = \frac{4\pi}{3} + n \cdot 2\pi \quad \text{tai} \quad x = -\frac{\pi}{3} + n \cdot 2\pi, \quad n \in \mathbb{Z}_+$$

$n$	$\frac{4\pi}{3} + n \cdot 2\pi$
0	$\frac{4\pi}{3} + 0 \cdot 2\pi = \frac{4\pi}{3}$ käy
1	$\frac{4\pi}{3} + 2\pi$ ei käy
2	—
-1	$-\frac{2\pi}{3}$ ei käy

$n$	$-\frac{\pi}{3} + n \cdot 2\pi$
0	$-\frac{\pi}{3}$ käy
1	$\frac{5\pi}{3}$ käy
2	$\frac{11\pi}{3}$ ei käy
3	$\frac{17\pi}{3}$ ei käy
-1	$-\frac{7\pi}{3}$ ei käy

$$\mathcal{V}: \frac{4\pi}{3}, -\frac{\pi}{3} \text{ ja } \frac{5\pi}{3}$$

Yhtälöt  $\sin x = \sin y$   
 $\cos x = \cos y$   
 $\tan x = \tan y$

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esim1  $\sin 2x = \sin \frac{\pi}{5}$

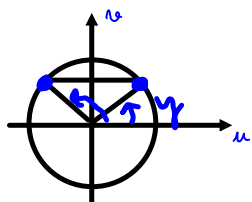
esim2  $\cos 2x = \frac{1}{\sqrt{2}}$

esim3  $\cos 3x = \cos 2x$

esim4  $-\sin 3x = \sin x$

esim  $\sin 2\alpha = \sin \frac{\pi}{5}$

$$\sin x = \sin y \Leftrightarrow x = y + n2\pi \text{ tai } x = \pi - y + n2\pi, \quad n \in \mathbb{Z}$$



$$2\alpha = \frac{\pi}{5} + n2\pi \quad \text{tai} \quad 2\alpha = \pi - \frac{\pi}{5} + n2\pi$$

$$\begin{array}{l} x = 2\alpha \\ y = \frac{\pi}{5} \end{array}$$

$$\alpha = \frac{\pi}{10} + n\pi \quad \text{tai} \quad 2\alpha = \frac{4\pi}{5} + n2\pi \quad | : 2$$

$$\alpha = \frac{4\pi}{10} + n\pi$$

$$\alpha = \frac{\pi}{10} + n\pi \quad \text{tai} \quad \alpha = \frac{2\pi}{5} + n\pi$$

huom!  
Myös  
jälkosa  
lasketon

$$\underline{\text{esim 3}} \quad \cos 3x = \cos 2x$$

$$\underline{\text{esim 4}} \quad -\sin 3x = \sin x$$

esim       $\sin 2x = \cos x$

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esim      $2 \Leftrightarrow x = -0,9$

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