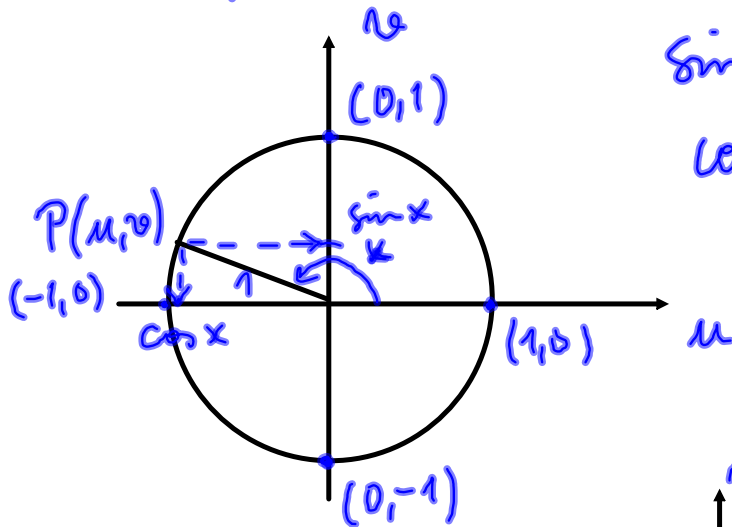
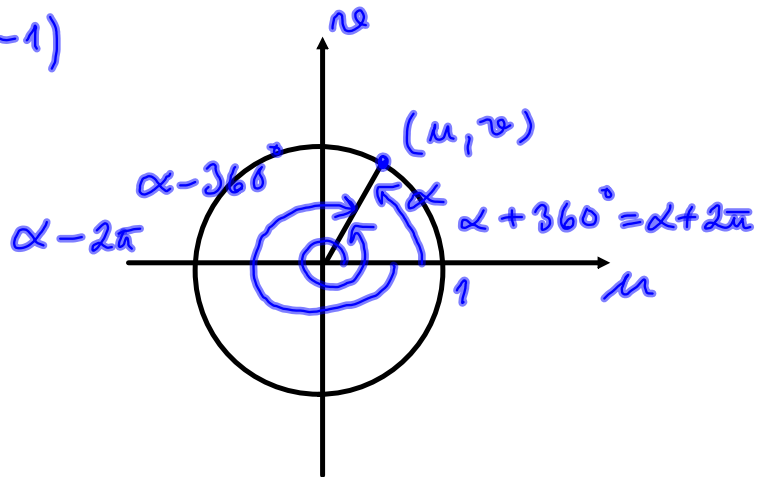


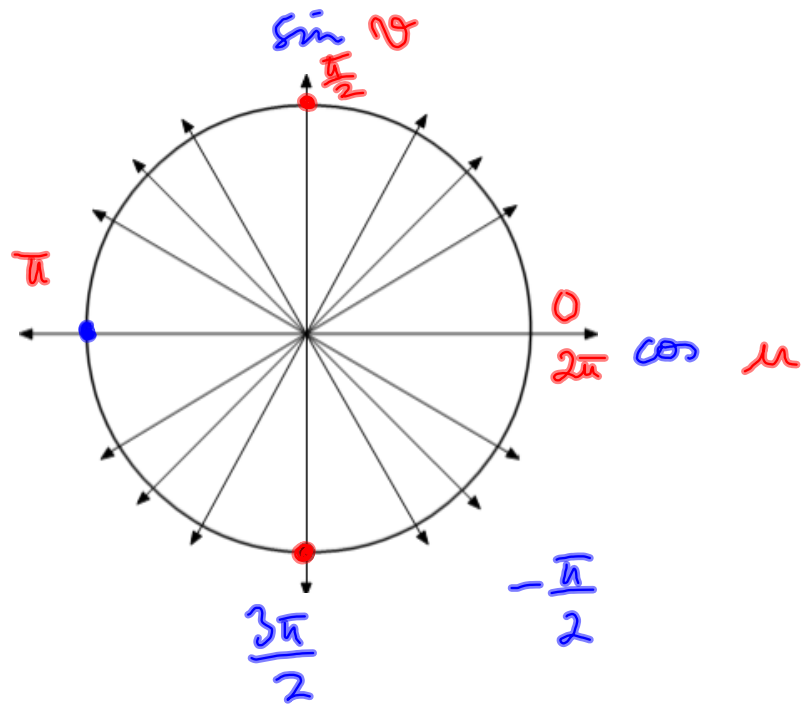
Kehäpiike



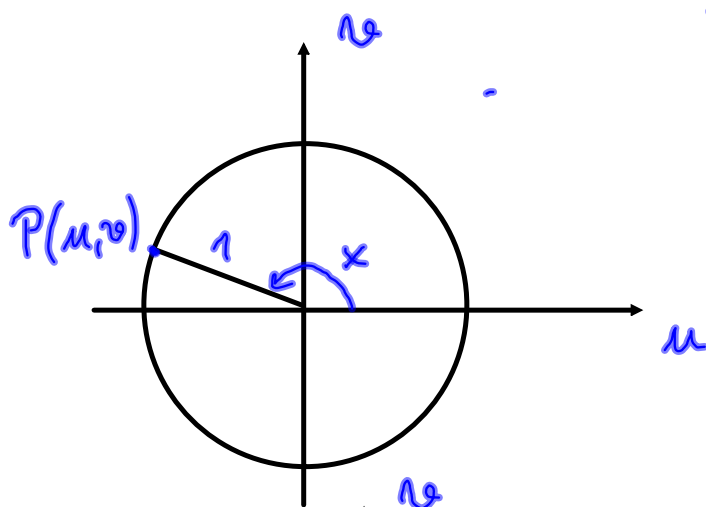
$$\sin x = v$$
$$\cos x = u$$



jaksollisuus
sinifunktio

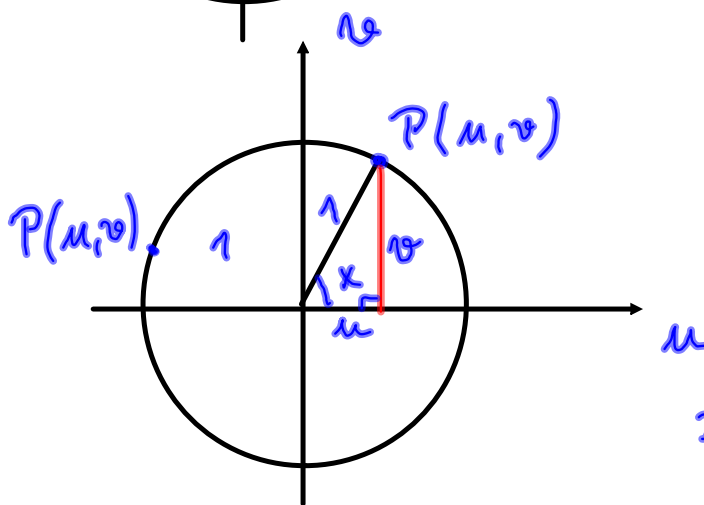


Funktion sini, kosini ja tangentti



$$\sin x = v$$

$$\cos x = u$$



$$\sin x = \frac{v}{1} = v$$

$$\cos x = \frac{u}{1} = u$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{v}{u},$$

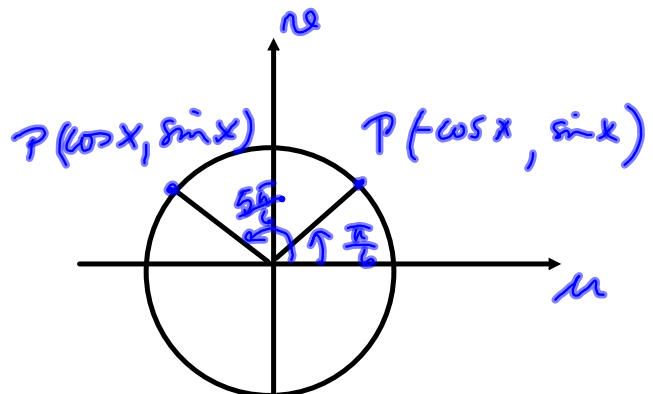
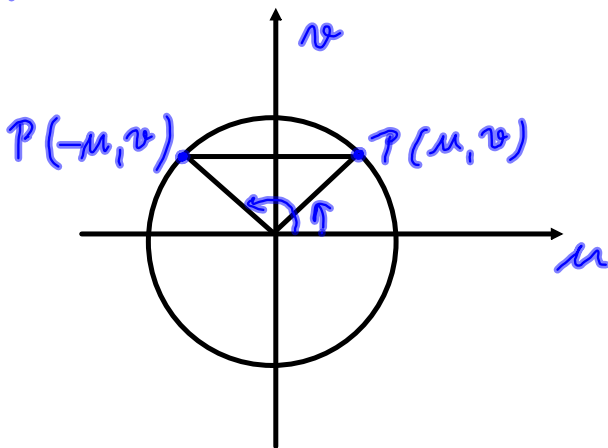
$$u \neq 0$$

Peruskaavat ja peruskaavat / self

Supplementtikulmat x ja $\pi - x$

Supplementtikulmien summa on $\pi = 180^\circ$.

ks. esim 2



kehäpiikeet
symmetriä
 π -akselin suhteen

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\tan(\pi - x) = -\tan x$$

esim

$$\begin{aligned} \sin(-105^\circ) &= \sin(180^\circ - (-105^\circ)) \\ &= \sin 285^\circ \\ &= -\frac{1}{4}(\sqrt{6} + \sqrt{2}) \end{aligned}$$

$$\sin \alpha = \sin(180^\circ - \alpha)$$

Kulman $x + \pi$ sini, kosini ja tangentti

$$\sin(x + \pi) = -\sin x$$

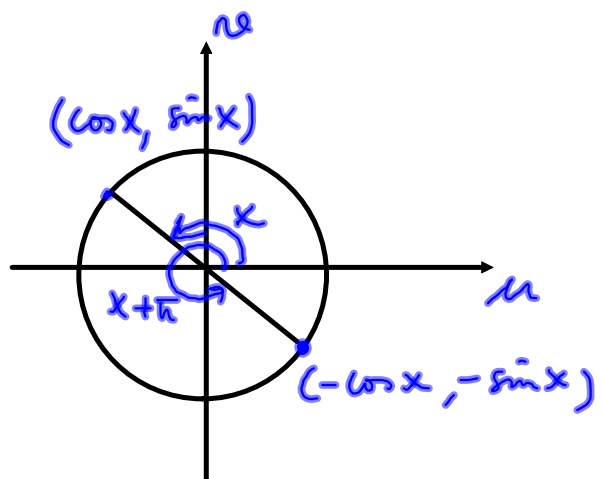
$$\cos(x + \pi) = -\cos x$$

$$\tan(x + \pi) = \tan x$$

(kann. tai väh.
puolikierrosta)
sin & kosini vastaluvut

esim 1/s. 23

esim 2/s. 24



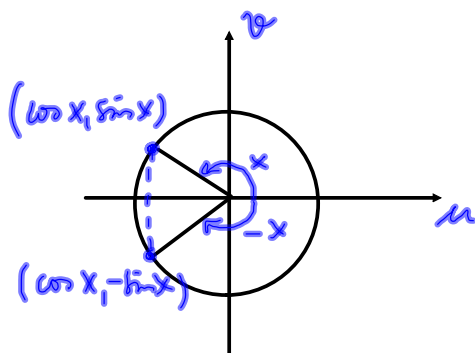
Vastakulmat x ja $-x$

esim 1 / s. 25

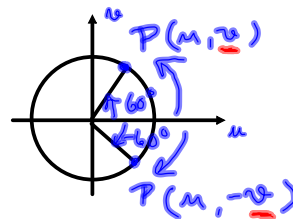
$$\sin(-x) = -\sin x \quad (\text{sini on pariton funktio})$$

$$\cos(-x) = \cos x \quad (\text{kosini on parillinen funktio})$$

$$\tan(-x) = -\tan x \quad (\text{tang. on pariton } \neq)$$



$$P(\cos \alpha, \sin \alpha) \\ = P(u, v)$$



esim a) $\sin(-60^\circ)$
 $= -\sin 60^\circ$
 $= -\frac{\sqrt{3}}{2}$

b) $\sin\left(-\frac{\pi}{6}\right)$
 $= -\sin\left(\frac{\pi}{6}\right)$
 $= -\frac{1}{2}$

c) $\cos \frac{11\pi}{3}$

c) $\cos \frac{11\pi}{3}$

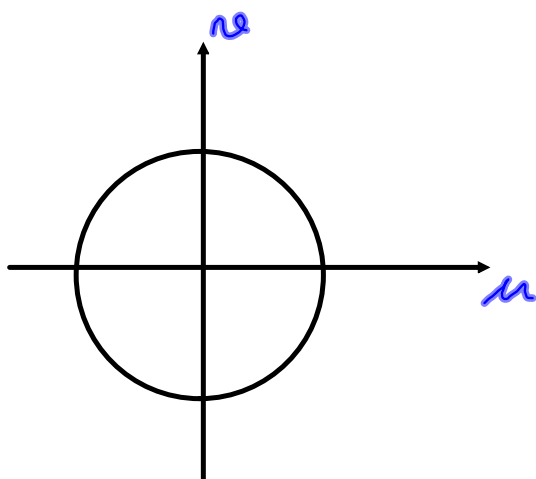
$$= \cos\left(3\frac{2}{3}\pi - 2 \cdot 2\pi\right)$$

$$= \cos\left(-\frac{\pi}{3}\right)$$

$$= \cos \frac{\pi}{3}$$

(kosini on parillinen)

$$= \underline{\underline{\frac{1}{2}}}$$



Peruskäyt

$$\sin^2 x + \cos^2 x = 1$$

$$\begin{aligned} (\sin x)^2 &= \sin^2 x \\ &= \sin x \cdot \sin x \end{aligned}$$

$$\tan x = \frac{\sin x}{\cos x}, \quad x \neq \frac{\pi}{2} + n\pi$$

esim Määritä lausekkeen $\cos \alpha$ tarkka arvo, kun

$$\sin \alpha = -\frac{1}{5} \text{ ja } \pi < \alpha < \frac{3\pi}{2}.$$

Ratk.
I tyyppi

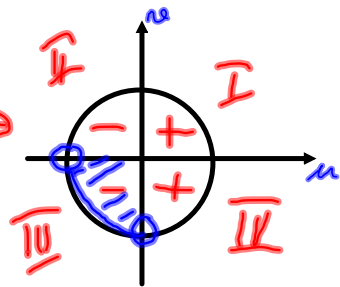
$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha \quad | \sqrt{}$$

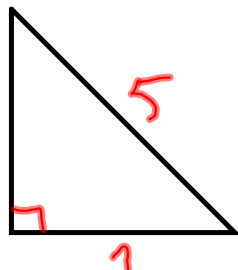
$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

$$= -\sqrt{1 - \left(-\frac{1}{5}\right)^2}$$

=



II tyyppi



$\sin \alpha$ Mietä sitä!

Komplementtikulman sini ja kosini

$$\sin x = \cos \left(\frac{\pi}{2} - x \right)$$

$$\cos x = \sin \left(\frac{\pi}{2} - x \right)$$

esim Määritä $\sin \left(x - \frac{\pi}{4} \right)$ kosinina avulla

$$\sin \left(x - \frac{\pi}{4} \right)$$

$$= \cos \left(\frac{\pi}{2} - \left(x - \frac{\pi}{4} \right) \right)$$

$$= \cos \left(\frac{\pi}{2} - x + \frac{\pi}{4} \right)$$

$$= \cos \left(\frac{3\pi}{4} - x \right)$$