

1.3. Laskukaavat

Yhteen- ja vähennyslaskukaavat

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Kaksinkertaisen kulman kaavat

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}, \text{ kun } x, 2x \neq \frac{\pi}{2} + k\pi$$

Esim Määritä lausekkeen

$\sin\left(2x + \frac{\pi}{6}\right)$ tarkka arvo, kun

$$\tan x = 2.$$

Ratk. $\sin\left(2x + \frac{\pi}{6}\right)$

$$= \sin 2x \cos \frac{\pi}{6} + \cos 2x \sin \frac{\pi}{6}$$

$$= 2 \sin x \cos x \cdot \frac{\sqrt{3}}{2} + (2 \cos^2 x - 1) \cdot \frac{1}{2}$$

$$= \sqrt{3} \sin x \cos x + \cos^2 x - \frac{1}{2}$$

$$= \frac{\sqrt{3} \sin x \cos^2 x}{\cos x} + \cos^2 x - \frac{1}{2}$$

$$= \sqrt{3} \tan x \cos^2 x + \cos^2 x - \frac{1}{2}$$

$$= \sqrt{3} \tan x \cdot \frac{1}{1 + \tan^2 x} + \frac{1}{1 + \tan^2 x} - \frac{1}{2}$$

$$\tan x = 2$$

lausekkeen
arvo $\sqrt{3} \cdot 2 \cdot \frac{1}{1+4} + \frac{1}{1+4} - \frac{1}{2} = \frac{2\sqrt{3}}{5} - \frac{3}{10}$

Pull

$$\cos^2 x = \frac{1}{1 + \tan^2 x}$$