

esim. Millä välin  $[0^\circ, 360^\circ[$   
kulmalla  $\beta$  on sama kehäpioste  
kuin kulmalle

- a)  $420^\circ$                       d)  $\frac{19\pi}{4}$   
 b)  $-150^\circ$   
 c)  $-2011^\circ$

Ratk. a)  $\beta = 420^\circ - 1 \cdot 360^\circ = 60^\circ$

b)  $\beta = -150^\circ + 360^\circ = 210^\circ$

c)  $\beta = -2011 + 6 \cdot 360^\circ = 149^\circ$

d)  $\frac{19\pi}{4}$

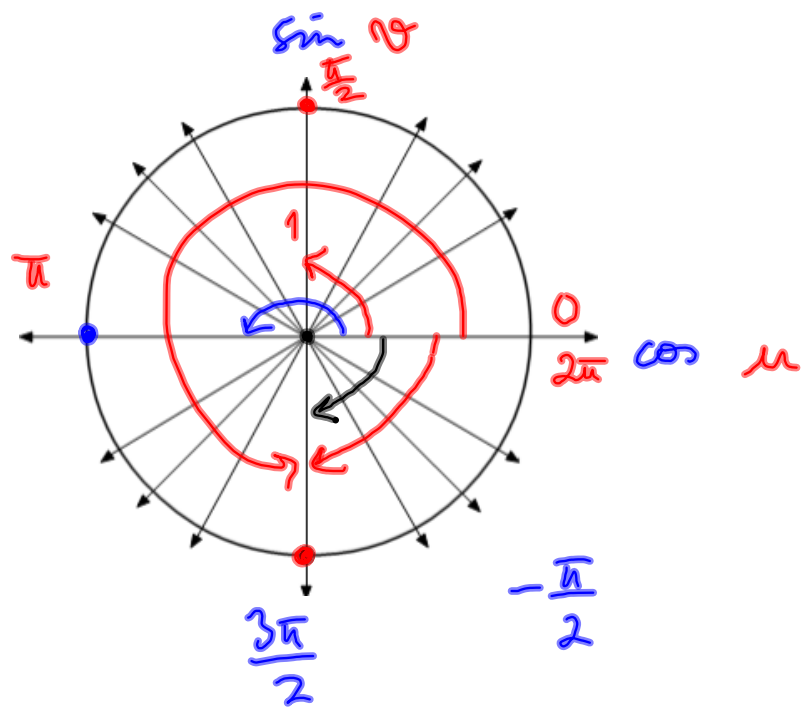
$$4 \frac{3}{4} \pi = \frac{3}{4} \pi + 4\pi = \frac{3}{4} \pi + 2 \cdot 2\pi$$

$$\beta = \frac{3}{4} \pi$$

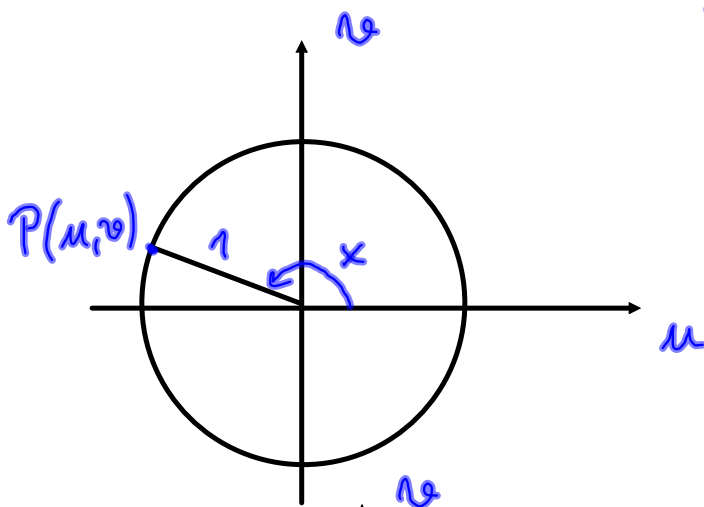
$$\sin \frac{\pi}{2} = 1$$

$$\cos \pi = -1$$

$$\cos \left(-\frac{\pi}{2}\right) = 0$$

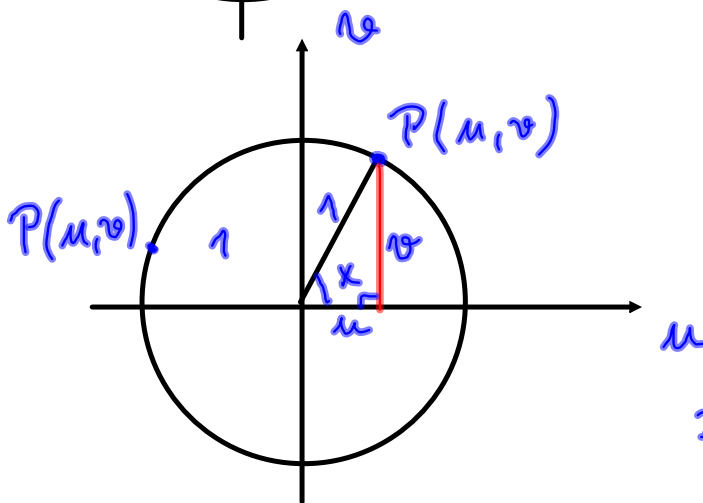


## Funktion sini, kosini ja tangentti



$$\sin x = v$$

$$\cos x = u$$

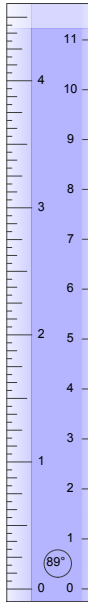


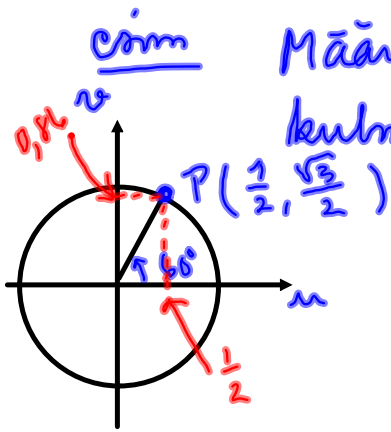
$$\sin x = \frac{v}{1} = v$$

$$\cos x = \frac{u}{1} = u$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{v}{u},$$

$$u \neq 0$$





cos Määritä kehäpiste, kun

kulma on

a)  $60^\circ$

b)  $\frac{2\pi}{3}$

kolit: ↷

laskin  
degree  
vai  
radianni

Ratk.  
a)

$$u = \cos 60^\circ = \frac{1}{2}$$

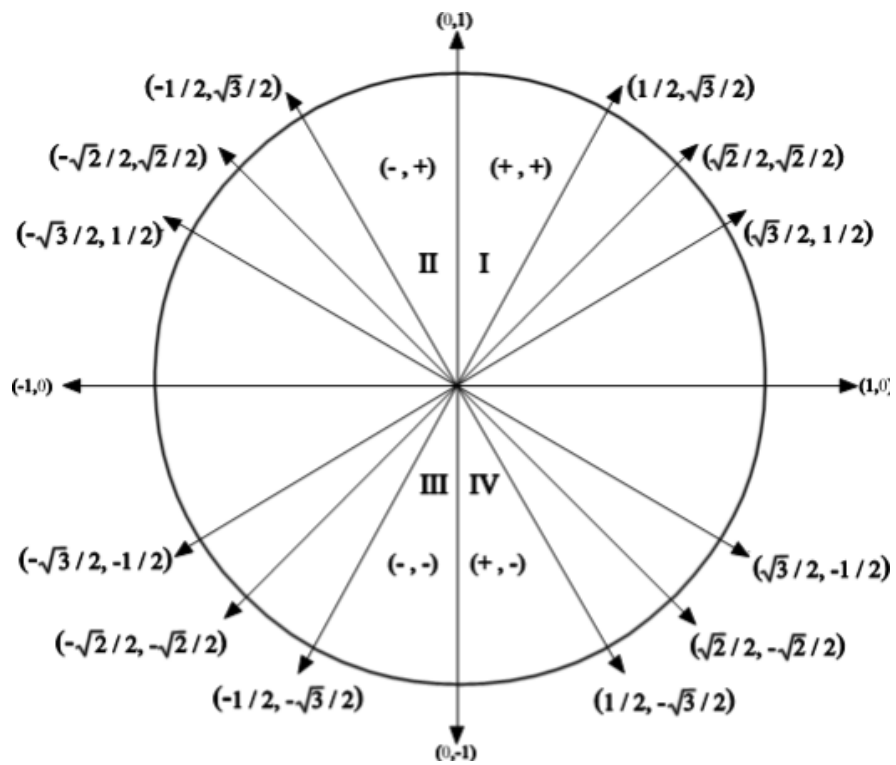
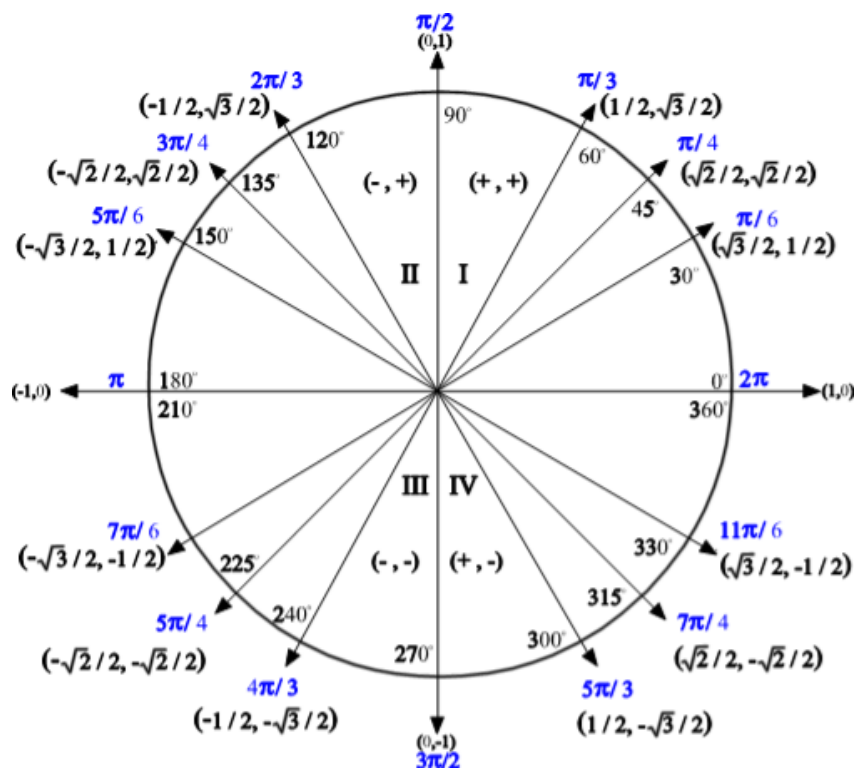
$$v = \sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0,86$$

kehäpiste on  $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

b)  $x = \cos \frac{2\pi}{3} = -\frac{1}{2}$

$$y = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

kp.  $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$



## Sinin ja kosinin tarkkoja arvoja

MAOL /s.

muistikaavat

esim. Mikä on kulman kehäpiste,  
kun kulma on  
a)  $60^\circ$                       b)  $\frac{2\pi}{3}$

Ratk.

$$x = \cos 60^\circ =$$

$$y = \sin 60^\circ =$$

## Määrittely- ja arvojoukot /s. 11

$$M_{\sin} = M_{\cos} = \mathbb{R}$$

$$M_{\tan} = \mathbb{R}, \text{ pois lukien } \cos x = 0 \text{ eli} \\ x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$$

$$A_{\sin} = A_{\cos} = [-1, 1]$$

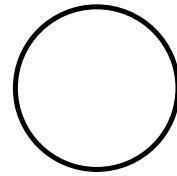
## Kosin ja väheneminen

### Merkit

### Jakosuhteisuus

erin Ratkaise yhtälö

$$\sin x = \frac{1}{2}$$



erin  $\cos x = -\frac{1}{2}$

## Sini- ja kosinifunktion jaksollisuus

$$x + n \cdot 2\pi, \quad n \in \mathbb{Z}$$

on sama kehäpiete kuin kulmalla  $x$ ,  
joten

$$\sin(x + n \cdot 2\pi) = \sin x$$

$$\cos(x + n \cdot 2\pi) = \cos x$$

-- pienintä positiivista jaksota  
nämöme PERUSJAKSOKSI



## Tangenttipiste

kulman  $x$  tangenttipiste on  $(1, \tan x)$ ,

kun  $x \neq \frac{\pi}{2} + n\pi$ ,  $n \in \mathbb{Z}$

- tangenttifunktion arvojoukko on  $\mathbb{R}$
- tangenttifunktio on jaksollinen, jakoa  $\pi$

Sektorin ala

$$A = \frac{1}{2} br$$

