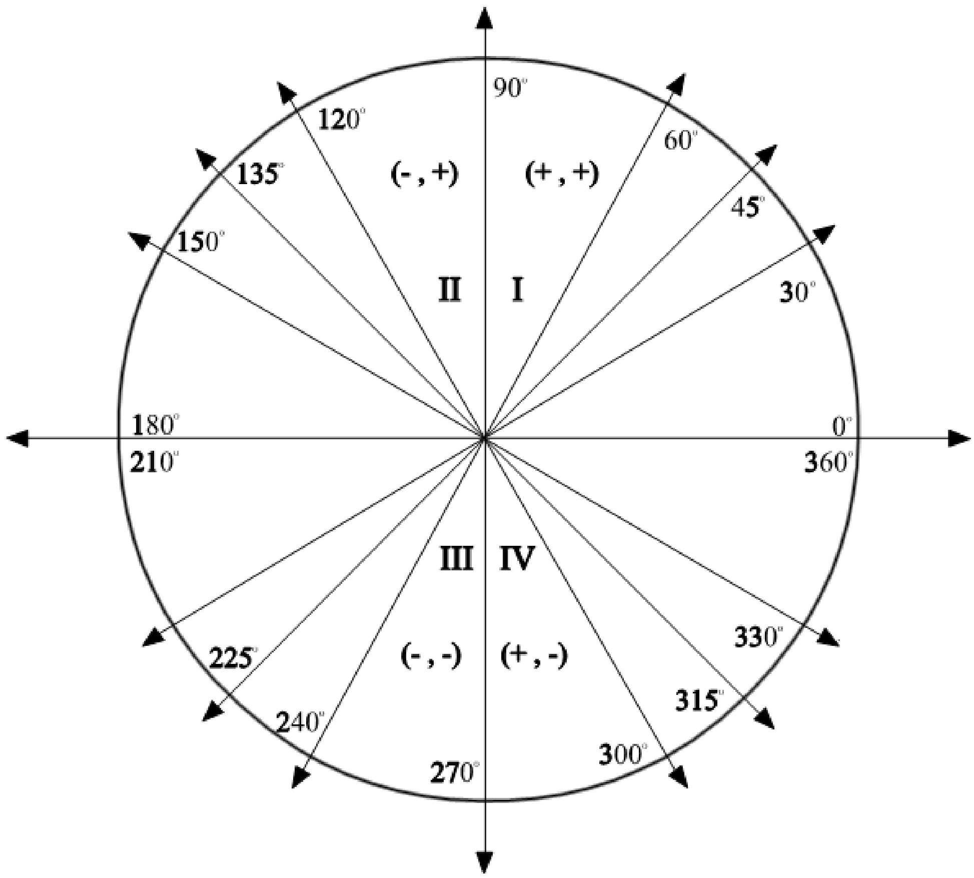
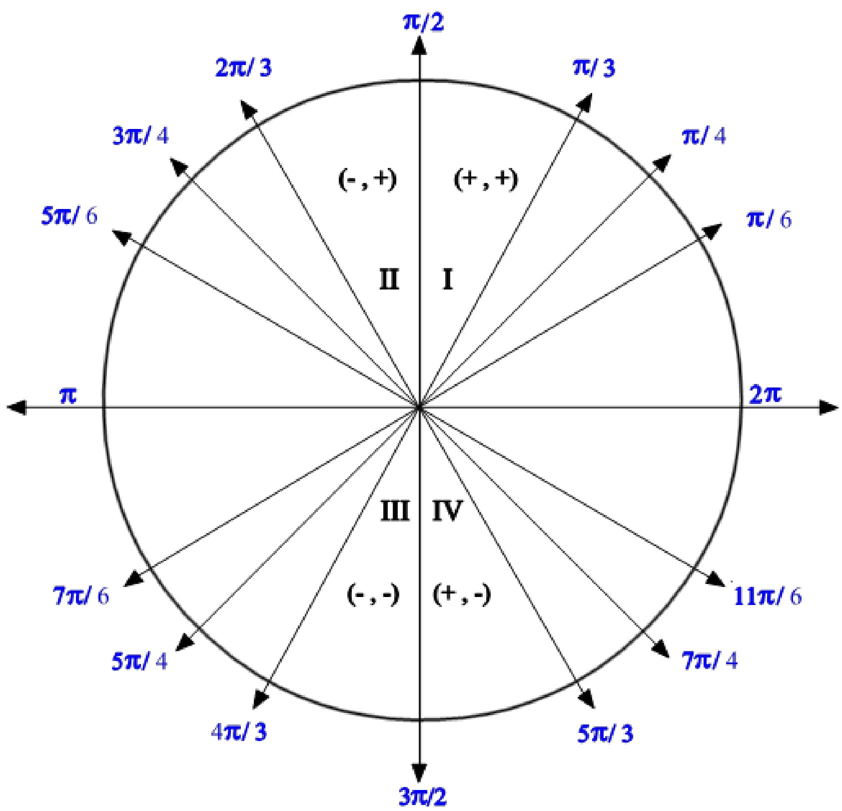
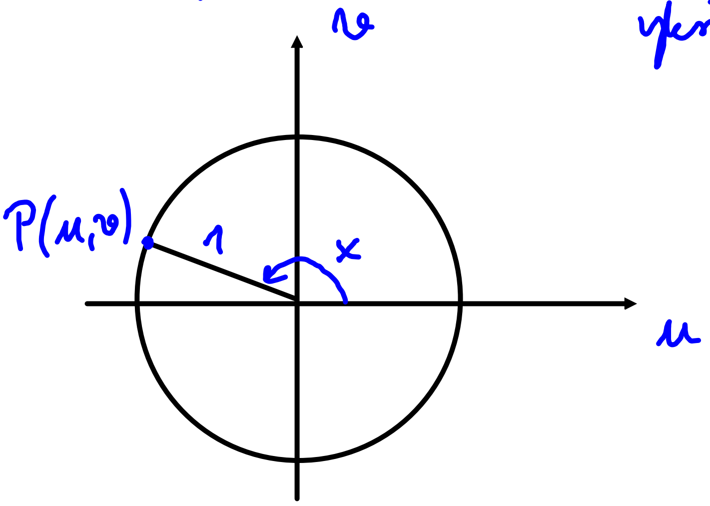

90°
120°
60°
135°
(- , +)
(+ , +)
45°
150°
30°
II
I
180°
0°
210°
360°
III
IV
330°
225°
(- , -)
(+ , -)
315°
240°
300°
270°


π/2
2π/ 3
π/ 3
3π/ 4
π/ 4
(- , +)
(+ , +)
5π/ 6
π/ 6
II
I
π
2π
III
IV
7π/ 6
(- , -)
(+ , -)
11π/ 6
5π/ 4
7π/ 4
4π/ 3
5π/ 3
3π/2

Kehäpiste



yksikköympyrä
$r = 1$

esim. Millä välin $[0^\circ, 360^\circ[$ kulmalla β on sama kehäpiste kuin kulmalla

a) 420°

b) -150°

c) -2011°

d) $\frac{19\pi}{4}$

Ratk. a) $\beta = 420^\circ - 1 \cdot 360^\circ = 60^\circ$

b) $\beta = -150^\circ + 360^\circ = 210^\circ$

c) $\beta = -2011 + 6 \cdot 360^\circ = 149^\circ$

d) $\frac{19\pi}{4}$

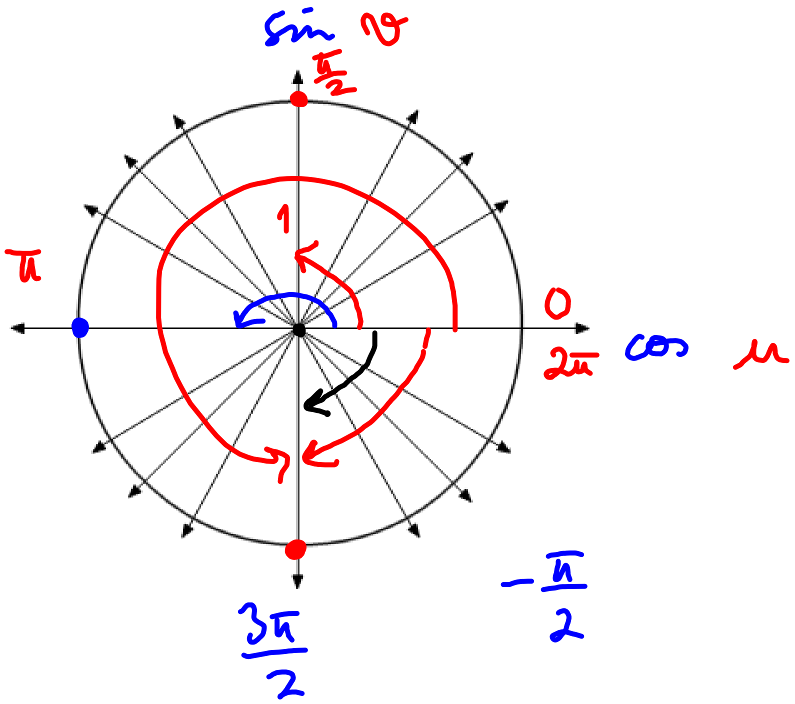
$$4\tfrac{3}{4}\pi = \tfrac{3}{4}\pi + 4\pi = \tfrac{3}{4}\pi + 2 \cdot 2\pi$$

$$\beta = \tfrac{3}{4}\pi$$

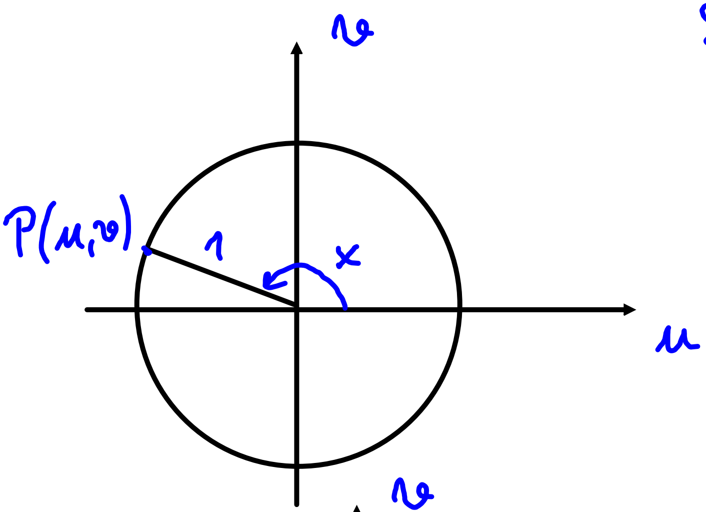
$\sin \frac{\pi}{2} = 1$

$\cos \pi = -1$

$\cos\left(-\frac{\pi}{2}\right) = 0$

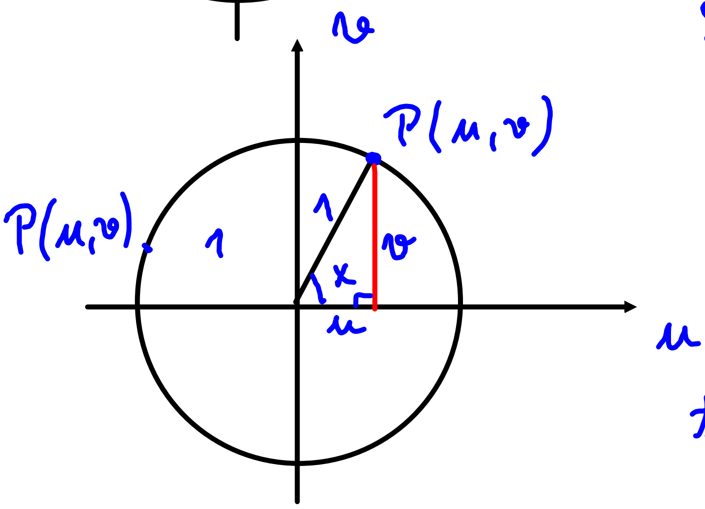


Funktion sini, kosini ja tangentti



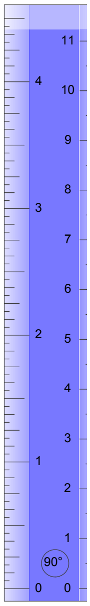
$\sin x = v$

$\cos x = u$



$\sin x = \frac{v}{1} = v$

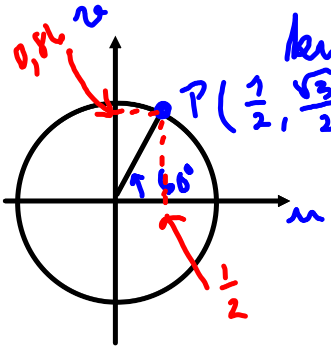
$\cos x = \frac{u}{1} = u$



$\tan x = \frac{\sin x}{\cos x} = \frac{v}{u}$,

$u \neq 0$

<u>esim</u> Määritä kehäpiste, kun kulma on



a) 60°

b) $\frac{2\pi}{3}$

kotit:

laskin degree vai radiaani

<u>Ratk.</u>

a) $u = \cos 60^\circ = \frac{1}{2}$

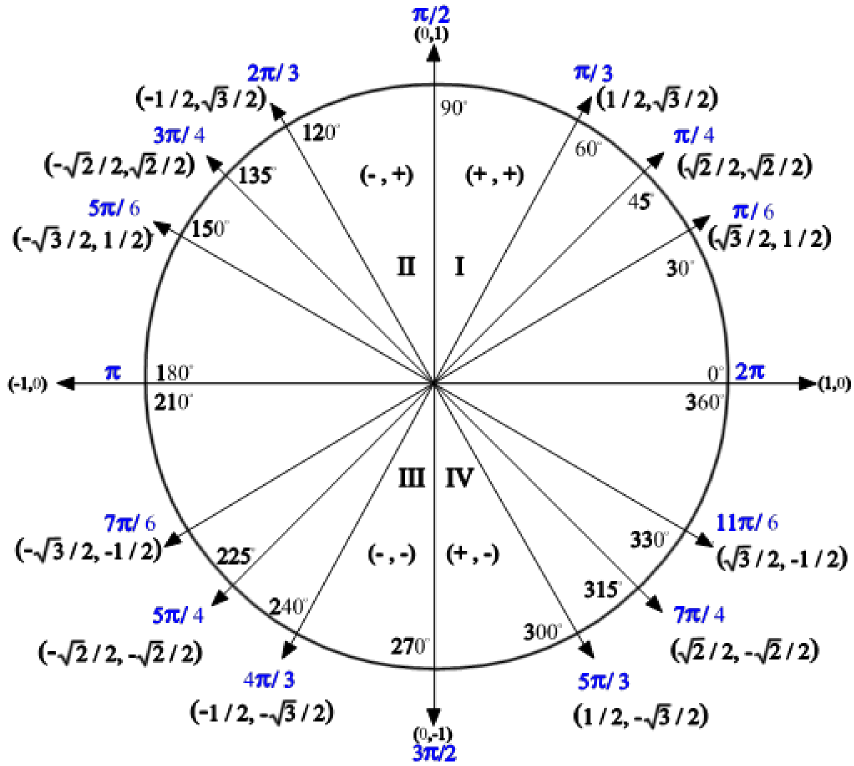
$v = \sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0{,}86$

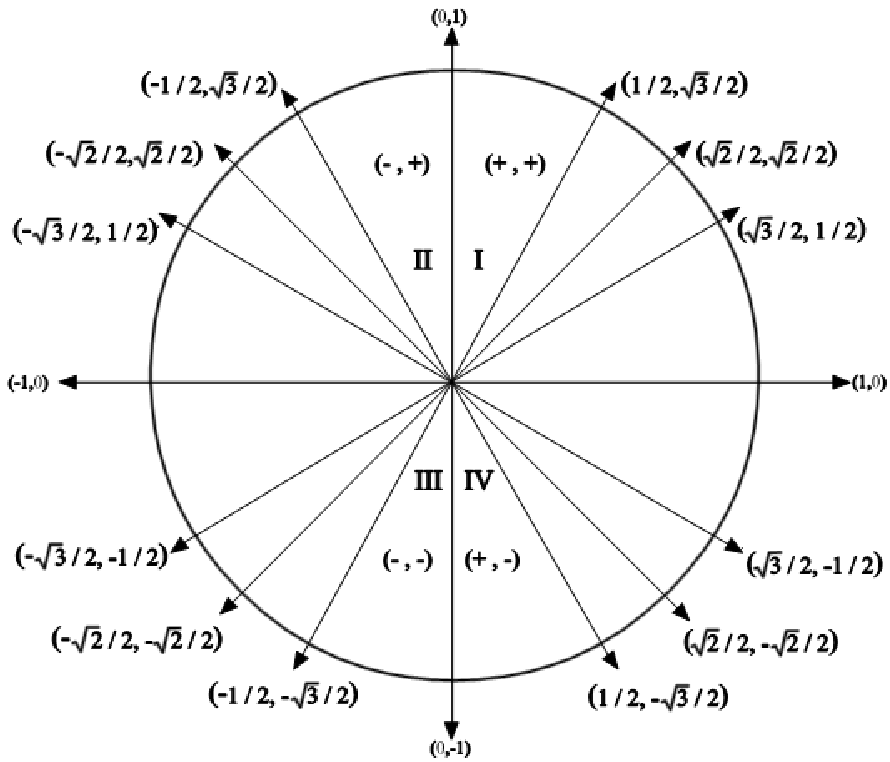
kehäpiste on $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $x = \cos \frac{2\pi}{3} = -\frac{1}{2}$

$y = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

kp. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$


π/2
(0,1)
2π/3
(-1/2, √3/2)
π/3
(1/2, √3/2)
3π/4
(-√2/2, √2/2)
π/4
(√2/2, √2/2)
5π/6
(-√3/2, 1/2)
π/6
(√3/2, 1/2)
90°
120°
60°
135°
45°
150°
30°
(-, +)
(+, +)
II
I
π
(-1,0)
180°
210°
0°
2π
(1,0)
360°
III
IV
(-, -)
(+, -)
7π/6
(-√3/2, -1/2)
11π/6
(√3/2, -1/2)
225°
330°
315°
5π/4
(-√2/2, -√2/2)
7π/4
(√2/2, -√2/2)
240°
270°
300°
4π/3
(-1/2, -√3/2)
5π/3
(1/2, -√3/2)
(0,-1)
3π/2


(0,1)
(-1/2, √3/2)
(1/2, √3/2)
(-√2/2, √2/2)
(√2/2, √2/2)
(-√3/2, 1/2)
(√3/2, 1/2)
(-, +)
(+, +)
II
I
(-1,0)
(1,0)
III
IV
(-√3/2, -1/2)
(√3/2, -1/2)
(-, -)
(+, -)
(-√2/2, -√2/2)
(√2/2, -√2/2)
(-1/2, -√3/2)
(1/2, -√3/2)
(0,-1)

Sinin ja kosinin tarkkoja arvoja

MAOL / s.

muistikaavat

esim Mikä on kulman kehäpiste, kun kulma on

a) 60° b) $\frac{2\pi}{3}$

Ratk.

$x = \cos 60^\circ =$

$y = \sin 60^\circ =$

Määrittely- ja arvojoukot / s. 11

$M_{\sin} = M_{\cos} = \mathbb{R}$

$M_{\tan} = \mathbb{R}$, poislukien $\cos x = 0$ eli $x = \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$

$A_{\sin} = A_{\cos} = [-1, 1]$

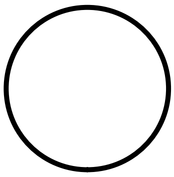
Kasvu ja väheneminen

Merkit

Jaksollisuus

<u>esim</u> Ratkaise yhtälö

$\sin x = \frac{1}{2}$



esim $\cos x = -\frac{1}{2}$

Sini- ja kosinifunktion jaksollisuus

$x + n \cdot 2\pi, \quad n \in \mathbb{Z}$

on sama kehäpiste kuin kulmalla x, joten

$\sin(x + n2\pi) = \sin x$

$\cos(x + n2\pi) = \cos x$

-- pienintä positiivista jaksoa sanomme PERUSJAKSOKSI

Tangenttipiste

kulman x tangenttipiste on $(1, \tan x)$,
kun $x \neq \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$

- tangenttifunktion arvojoukko on $\mathbb{R}$
- tangenttifunktio on jaksollinen, jakso π

Sektorin ala

$$A = \frac{1}{2} b r$$