

DERIVOINTI – JA INTEGROINTIKERTAUS

Derivoi seuraavat funktiot

1. **a)** $f(x) = x^{10}$
 b) $f(x) = 6x^3 - x - e^x - e^2$
 c) $f(x) = \cos 2x$
 d) $f(x) = e^{-x^2}$

2. **a)** $f(x) = x^3 \cdot e^x$
 b) $f(x) = \ln x$
 c) $f(x) = \ln(3x + 1)$
 d) $f(x) = (x^2 - 1)^3(1 - x)$

3. **a)** $f(x) = \frac{x^2 + x}{2x^2 + 1}$
 b) $f(x) = \sqrt[3]{x^2}$

 c) $f(x) = \sqrt{-x} - \sqrt{1 + 2x}$
 d) $f(x) = \sqrt{1 - x^2}$

4. **a)** $f(x) = \sin \frac{x}{3}$
 b) $f(x) = \frac{2}{3} \cos 7x$
 c) $f(x) = \sin^2(1 - \pi x)$
 d) $f(x) = \sin(3x - 2)^2$

5. **a)** $\int 5x^2 dx$

 b) $\int (t + \cos x) dx$

 c) $\int x \sin t dx$
 d) $\int x \sin t dt$

Integroi seuraavat funktiot

7. **a)** $f(x) = (5x - 2)^6$
 b) $f(x) = \frac{1}{1 - x}$
 c) $f(x) = \frac{1}{x \ln x}$
 d) $f(x) = \frac{\ln x}{x}$

6. **a)** $f(x) = 7x^3 - \frac{1}{2}x^2 + 3$
 b) $f(x) = \sqrt[5]{x}$
 c) $f(x) = \frac{4}{x^4}$
 d) $f(x) = x^{-1}$

8. **a)** $f(x) = 2^{-3x}$
 b) $f(x) = \cos 2x$
 c) $f(x) = \sin^2 x \cos x$
 d) $f(x) = \tan x$

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2. a) $D x^{10} = 10x^9$

b) $D (6x^3 - x - e^x - e^2) = 18x^2 - 1 - e^x$

c) $D \cos 2x = -2 \sin 2x$

d) $D e^{x^2} = e^{x^2} \cdot D(x^2) = 2xe^{x^2}$

2. a) $D(x^3 * e^x) = 3x^2 \cdot e^x + x^3 \cdot e^x = e^x x^2 (x+3)$

b) $D \ln x = \frac{1}{x}, \quad x > 0$

c) $D \ln(3x+1) = \frac{1}{3x+1} D(3x+1) = \frac{3}{3x+1}, \quad x > -\frac{1}{3}$

d) $D(x^2 - 1)^3(1 - x)$

$$= 3(x^2 - 1)^2 \cdot 2x \cdot (1 - x) + (x^2 - 1)^3(-1)$$

$$= (x^2 - 1)^2 (6x - 6x^2 - x^2 + 1) = (x^2 - 1)^2 (-7x^2 + 6x + 1)$$

3. a) $D \frac{x^2 + x}{2x^2 + 1} = \frac{(2x+1)(2x^2+1) - (x^2+x) \cdot 4x}{(2x^2+1)^2} = \frac{-2x^2 + 2x + 1}{4x^4 + 4x^2 + 1}$

b) $D \sqrt[3]{x^2} = D x^{\frac{2}{3}} = \frac{2}{3} x^{\frac{2}{3}-1} = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3} \cdot \frac{1}{x^{\frac{1}{3}}} = \frac{2}{3\sqrt[3]{x}} = \frac{2(\sqrt[3]{x})^2}{3x}, \quad x \neq 0$

c)

$$D(\sqrt{-x} - \sqrt{1+2x}) = D((-x)^{\frac{1}{2}} - (1+2x)^{\frac{1}{2}}) = \frac{1}{2}(-x)^{-\frac{1}{2}} \cdot (-1) - \frac{1}{2}(1+2x)^{-\frac{1}{2}} \cdot 2$$

$$= -\frac{\sqrt{-x}}{2\sqrt{-x}} - \frac{\sqrt{1+2x}}{\sqrt{1+2x}} = \frac{\sqrt{-x}}{2x} - \frac{\sqrt{1+2x}}{1+2x}, \quad -\frac{1}{2} \leq x \leq 0$$

d) $D\sqrt{1-x^2} = D(1-x^2)^{\frac{1}{2}} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = -\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} \cdot x = \frac{x\sqrt{1-x^2}}{1-x^2}, \quad -1 < x < 1$

4. a) $D \sin \frac{x}{3} = \frac{1}{3} \cos \frac{x}{3}$

b) $D \frac{2}{3} \cos 7x = \frac{2}{3} (-\sin 7x) \cdot 7 = -\frac{14}{3} \sin 7x$

c) $D \sin^2(1 - \pi x) = 2 \sin(1 - \pi x) \cos(1 - \pi x)(-\pi)$

$$= -\pi 2 \sin(1 - \pi x) \cos(1 - \pi x) = -\pi \sin 2(1 - \pi x) = -\pi \sin(2 - 2\pi x)$$

d) $D \sin(3x-2)^2 = \cos(3x-2)^2 D(3x-2)^2 = (18x-12) \cos(3x-2)^2$

5. a) $\int 5x^2 dx = \frac{5}{3}x^3 + C$

b) $\int (t + \cos x) dx = tx + \sin x + C$

c) $\int x \sin t dx = \sin t \int x dx = \frac{1}{2}x^2 \sin t + C$

d) $\int x \sin t dt = x \int \sin t dt = x(-\cos t) + C = -x \cos t + C$

6. a) $\int (7x^3 - \frac{1}{2}x^2 + 3) dx = \frac{7}{4}x^4 - \frac{1}{2} \cdot \frac{1}{3}x^3 + 3x + C = \frac{7}{4}x^4 - \frac{1}{6}x^3 + 3x + C$

b) $\int \sqrt[5]{x} dx = \int x^{\frac{1}{5}} dx = \frac{5}{6}x^{\frac{6}{5}} + C = \frac{5}{6}x^{\frac{6}{5}} + C$

c) $\int \frac{4}{x^4} dx = 4 \int x^{-4} dx = 4 \cdot (-\frac{1}{3}x^{-3}) + C = -\frac{4}{3x^3} + C, x \neq 0$

d) $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C, x \neq 0$

7. a) $\int (5x-2)^6 dx = \frac{1}{5} \int 5 \cdot (5x-2)^6 dx = \frac{1}{5} \cdot \frac{1}{7} (5x-2)^7 + C = \frac{1}{35} (5x-2)^7 + C$

b) $\int \frac{1}{1-x} dx = -\int \frac{-1}{1-x} dx = -\ln|1-x| + C, x \neq 1$

c) $\int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln|\ln x| + C, x > 0, x \neq 1$

d) $\int \frac{\ln x}{x} dx = \int \ln x D(\ln x) dx = \frac{1}{2}(\ln x)^2 + C, x > 0, x \neq 1$

8. a)

$$\int 2^{-3x} dx = (-\frac{1}{3}) \int (-3) \cdot 2^{-3x} dx = (-\frac{1}{3}) \cdot \frac{2^{-3x}}{\ln 2} + C = -\frac{1}{2^{3x} \cdot 3 \ln 2} = -\frac{1}{2^{3x} \ln 8}, (a > 0, a \neq 1)$$

b) $\int \cos 2x dx = \frac{1}{2} \int 2 \cos 2x dx = \frac{1}{2} \sin 2x + C$

c) $\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x + C$

d) $\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$