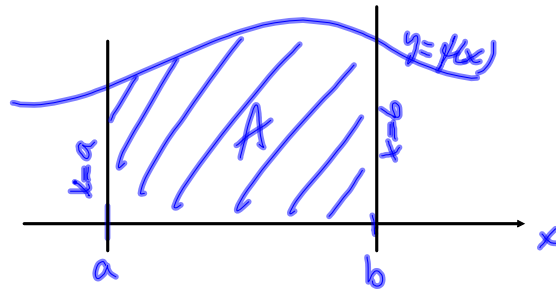
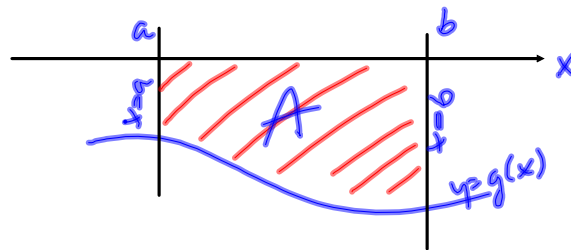


### 3 SOVELLUKSIA



$$A = \int_a^b f(x) dx, \quad f(x) \geq 0, \\ \text{kun } x \in [a, b]$$



$$A = - \int_a^b g(x) dx, \quad g(x) \leq 0, \quad x \in [a, b]$$

esim. laske käyrän  $y=2x-x^2$  ja x-akselin rajoittaman alueen ala.

Ratk. (laske ensin apuvälilinen, kotoon kurot käsi)

$$\text{ml: } 2x - x^2 = 0$$

kuro  
seur. diaari

$$x(2-x) = 0$$

$$x=0 \text{ tai } 2-x=0$$

$$x=0 \text{ tai } x=2$$

$$\int_0^2 (2x - x^2) dx$$

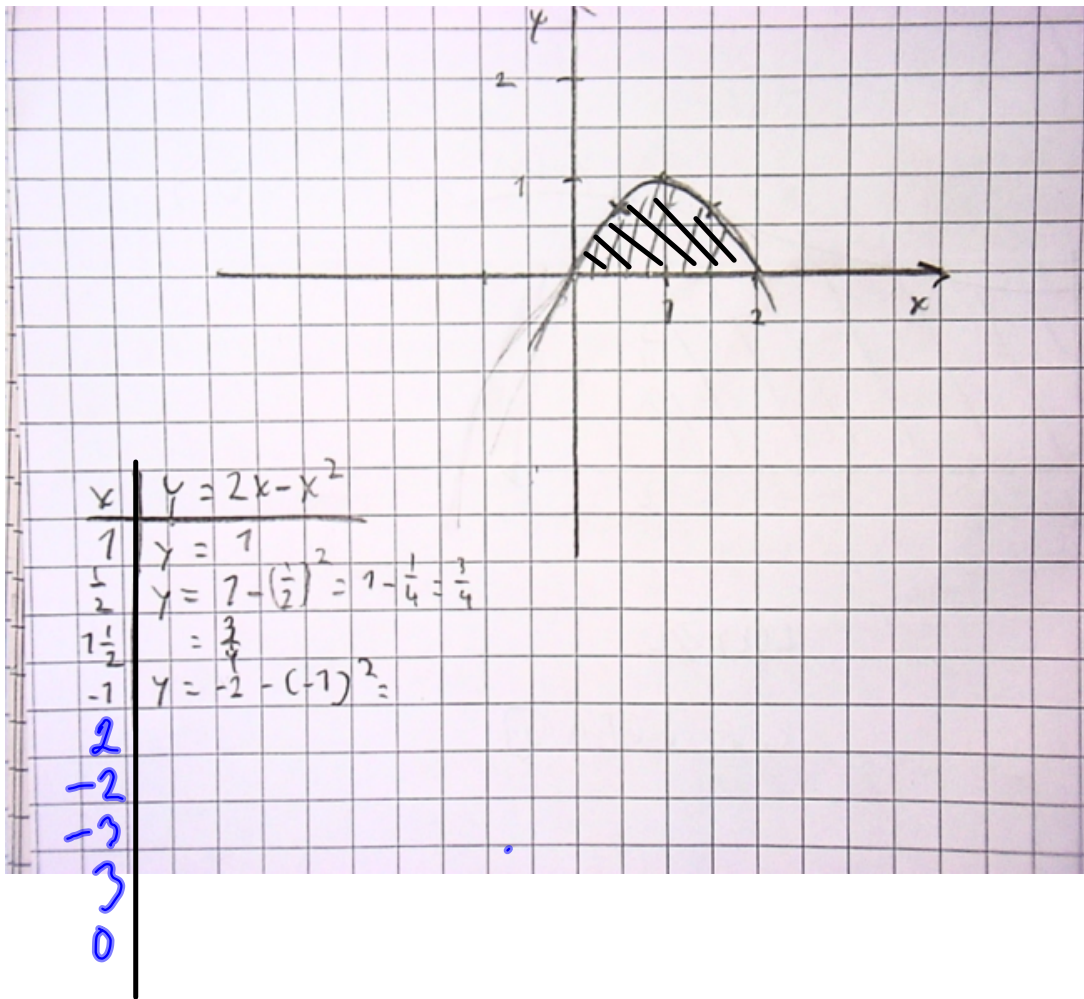
kuro

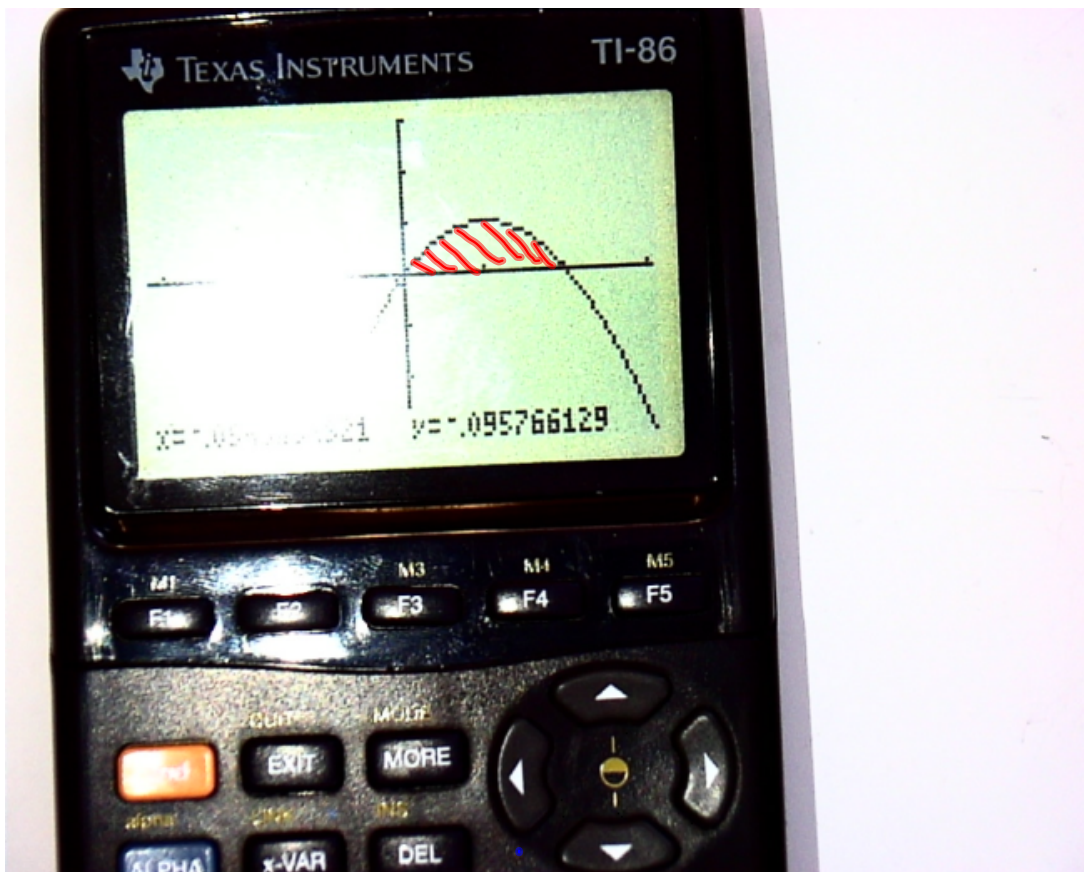
$$= \int_0^2 \left( x^2 - \frac{1}{3}x^3 \right)$$

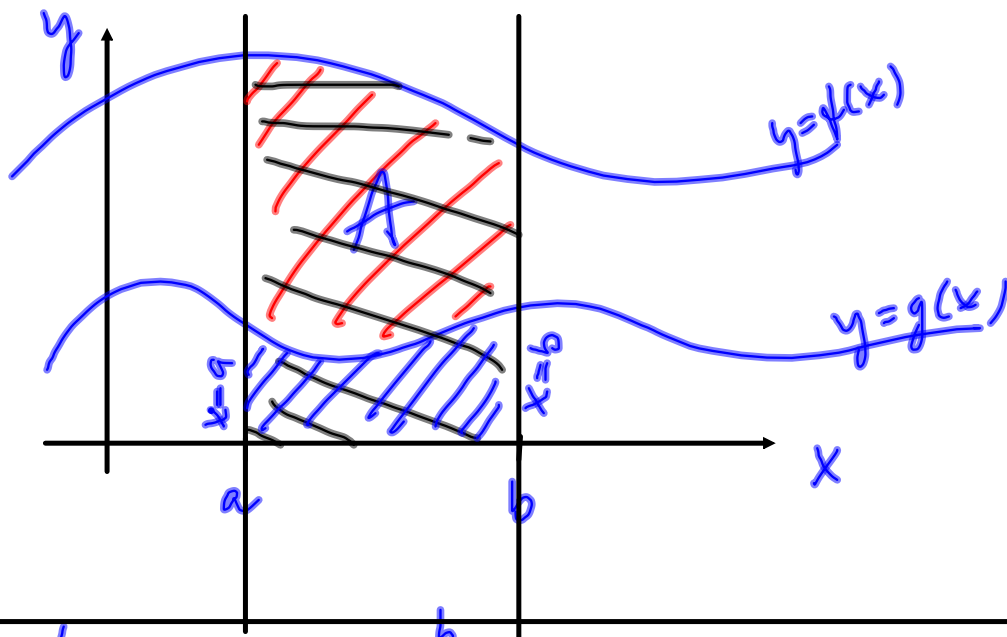
$$= \left( 2^2 - \frac{1}{3} \cdot 2^3 \right) - \left( 0^2 - \frac{1}{3} \cdot 0^3 \right)$$

$$= \frac{4}{3}$$

$$x \mid y = 2x - x^2$$







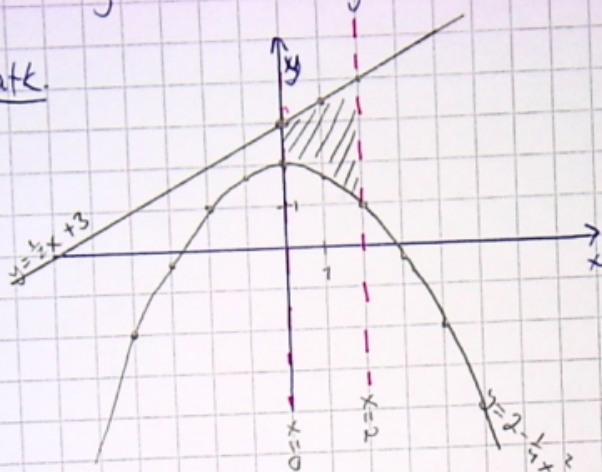
$$A = \int_a^b f(x) dx - \int_a^b g(x) dx$$

$$= \int_a^b (f(x) - g(x)) dx, \quad f(x) \geq g(x), \quad x \in [a, b]$$

esim 2 laske käyriä  $y = 2 - \frac{1}{4}x^2$ ,  
 $y = \frac{1}{2}x + 3$ ,  $x = 0$  ja  $x = 2$   
 rajoittaman alueen ala.

Ratk.

Ratk.



$$\begin{aligned} & \int_0^2 \left( \left( \frac{1}{2}x + 3 \right) - \left( 2 - \frac{1}{4}x^2 \right) \right) dx \\ &= \int_0^2 \left( \frac{1}{2}x + 3 - 2 + \frac{1}{4}x^2 \right) dx \\ &= \int_0^2 \left( \frac{1}{2}x + 1 + \frac{1}{4}x^2 \right) dx \\ &= \int_0^2 \left( \frac{1}{2}x^2 + \frac{1}{4}x^2 + x \right) dx \\ &= \left( \frac{1}{12}x^3 + \frac{1}{4}x^2 + 2x \right) \Big|_0^2 \\ &= \left( \frac{1}{12} \cdot 2^3 + \frac{1}{4} \cdot 2^2 + 2 \right) - \left( \frac{1}{12} \cdot 0^3 + \frac{1}{4} \cdot 0^2 + 0 \right) \\ &= \frac{8}{12} + 1 + 2 \\ &= \frac{31}{6} \end{aligned}$$

## Käyrän ja y-akselin rajaama alue

esim 3 Määritä funktioiden  $y = x^2$ , ( $x \geq 0$ ),  $y = 1$ ,  
 $y = 4$  ja  $x = 0$  rajaaman alueen ala.

Ratk. Pinta-alkion ala on  $x dy = dA$

$$y = x^2 \quad | \sqrt{\quad}$$

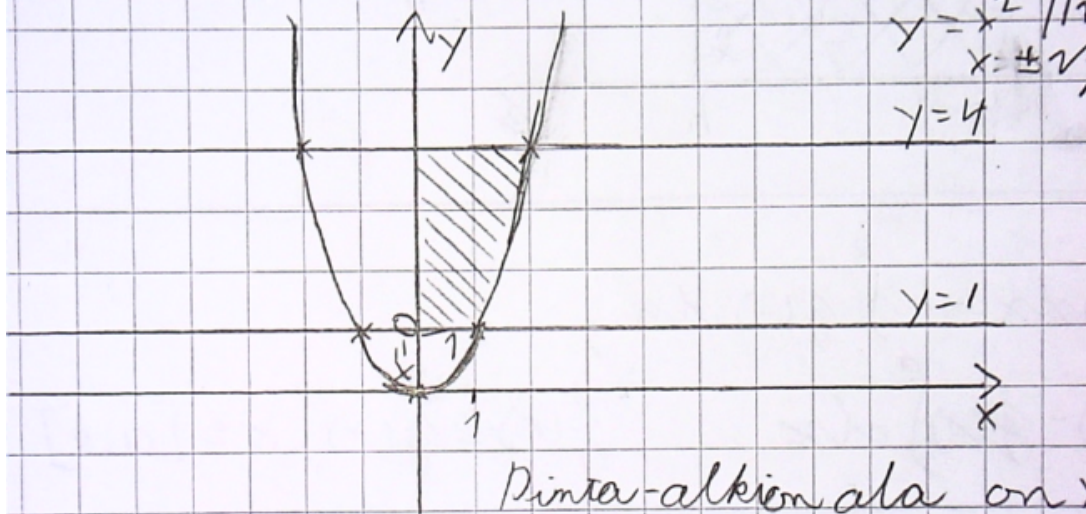
$$x = \sqrt{y}$$

$$A = \int_1^4 dA = \int_1^4 x dy = \int_1^4 \sqrt{y} dy = \int_1^4 y^{\frac{1}{2}} dy$$

=

$$= 4 \frac{2}{3}$$

$y=4$  ja  $x=0$  rajaaman ala



$$A = \int_0^2$$