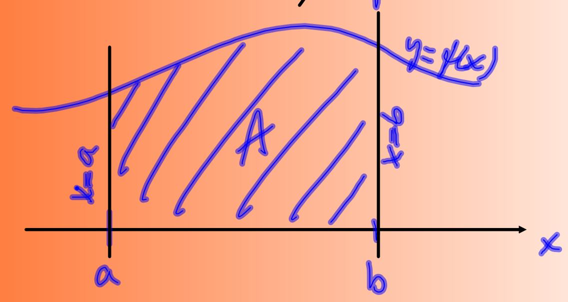
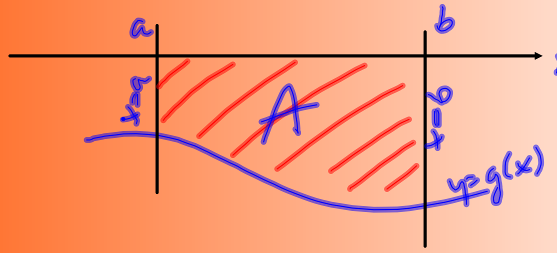
3 SOVELLUKSIA / slf



$$A = \int_a^b f(x)\,dx \quad , \quad f(x) \geq 0,$$

kun $x \in [a, b]$



$$A = -\int_a^b g(x)\,dx \quad , \quad g(x) \leq 0, \quad x \in [a, b]$$

Esim. Laske käyrän $y = 2x - x^2$ ja x-akselin rajoittaman alueen ala.

Ratk. (Laskin apuvälineenä, kotona kuvio käsin)

kuvio seur. diassa

nk: $2x - x^2 = 0$

$x(2 - x) = 0$

$x = 0$ tai $2 - x = 0$

$x = 0$ tai $x = 2$

$$\int_0^2 (2x - x^2)\,dx$$

kuvio

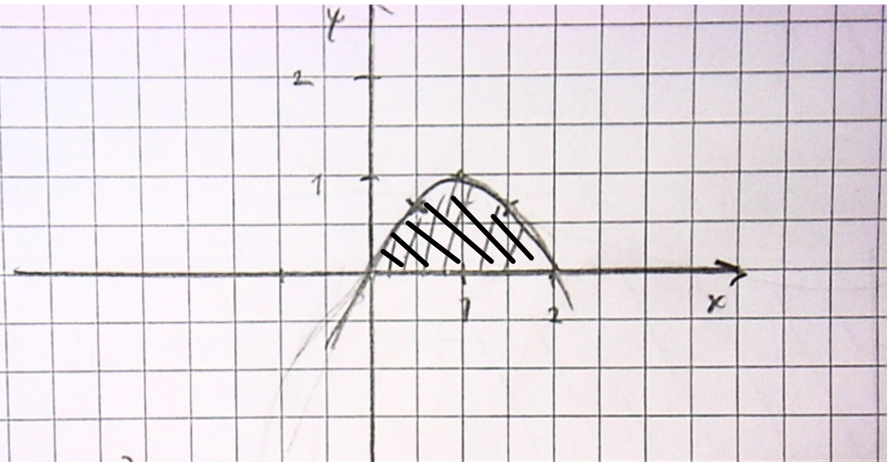
$$= \Big/_0^2 \left(x^2 - \frac{1}{3}x^3\right)$$

$$= \left(2^2 - \frac{1}{3}\cdot 2^3\right) - \left(0^2 - \frac{1}{3}\cdot 0^3\right)$$

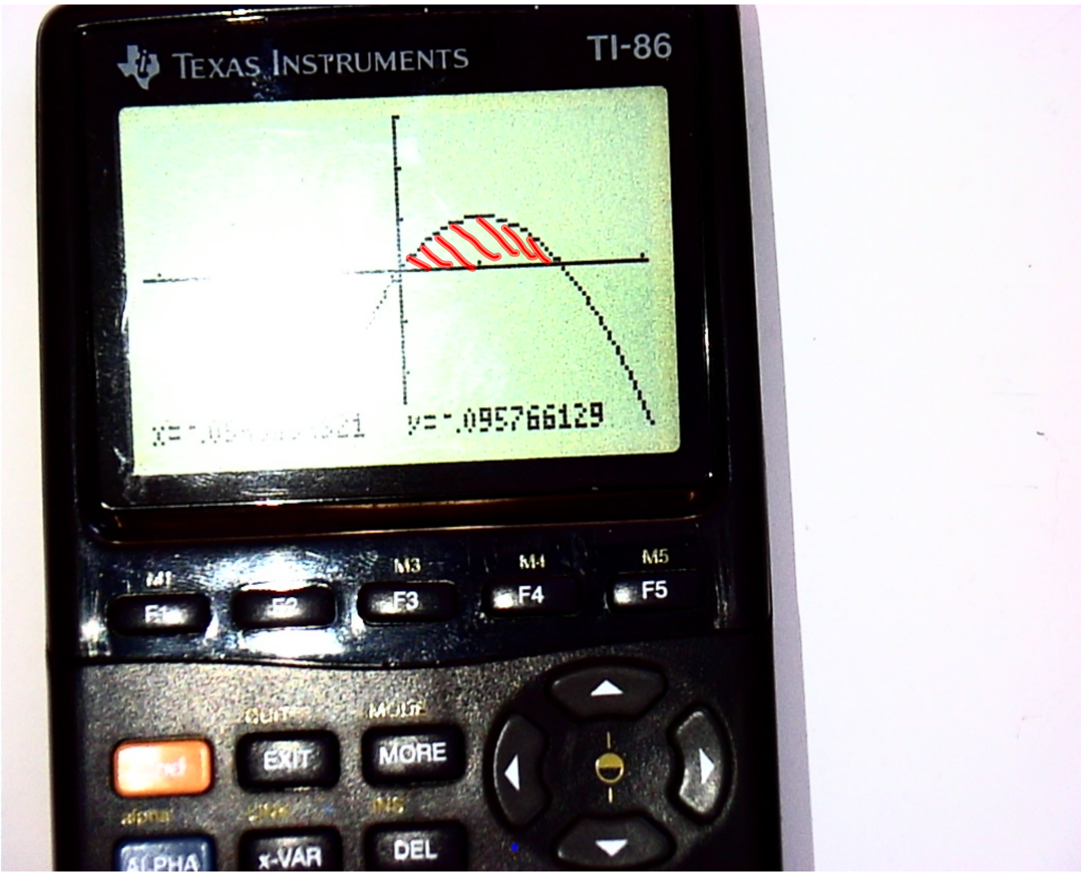
$$= \frac{4}{3}$$

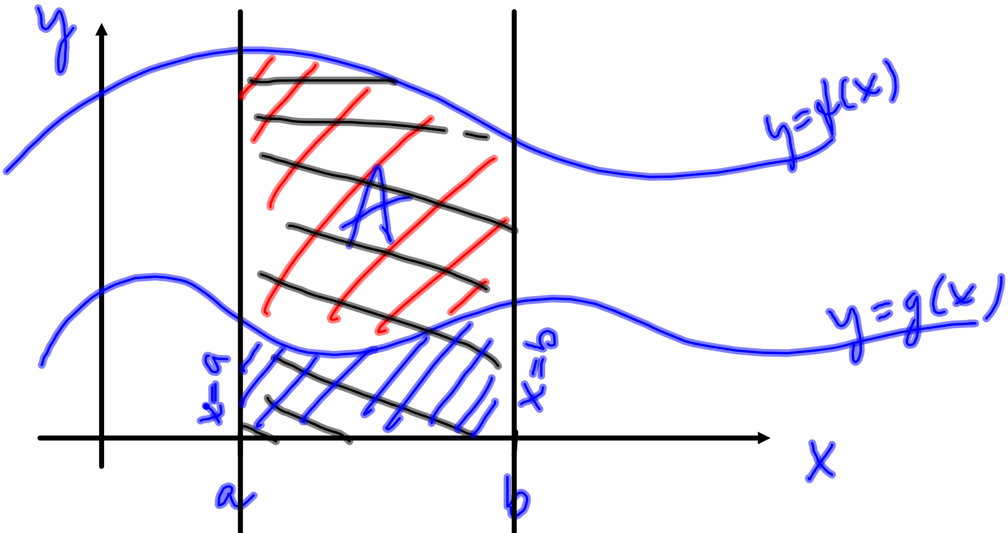
x	$y=2x-x^2$

Matta-teoriaa pinta-alan laskemisesta



x	$y = 2x - x^2$
1	$y = 1$
$\frac{1}{2}$	$y = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$
$1\frac{1}{2}$	$= \frac{3}{4}$
-1	$y = -2 - (-1)^2 =$
2	
-2	
-3	
3	
0	


TEXAS INSTRUMENTS
TI-86
y=-.095766129
M3
M4
M5
F3
F4
F5
EXIT
MORE
x-VAR
DEL



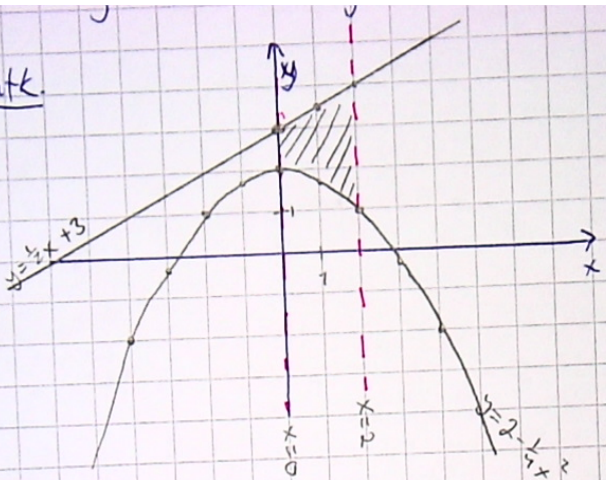
$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b \big(f(x) - g(x)\big)\,dx \quad , \quad f(x) \geq g(x), \quad x \in [a, b]$$

<u>esim 2</u> Laske käyrien $y = 2 - \frac{1}{4}x^2$, $y = \frac{1}{2}x + 3$, $x = 0$ ja $x = 2$ rajoittaman alueen ala.

<u>Ratk.</u>

Ratk.



$$\int_0^2 \left(\left(\frac{1}{2}x+3\right)-\left(2-\frac{1}{4}x^2\right)\right)$$

$$= \int_0^2 \left(\underbrace{\frac{1}{2}x+\overbrace{3-2}^{1}+\frac{1}{4}x^2}_{\frac{1}{4}x^2+\frac{1}{2}x+1}\right)$$

$$= \Big/_0^2 \left(\frac{1}{3}\cdot\frac{1}{4}x^3+\frac{1}{2}\cdot\frac{1}{2}x^2+x\right)$$

$$= \Big/_0^2 \left(\frac{1}{12}x^3+\frac{1}{4}x^2+x\right)$$

$$= \left(\frac{1}{12}\cdot 2^3+\frac{1}{4}\cdot 2^2+2\right)-\left(\frac{1}{12}\cdot 0^3+\frac{1}{4}\cdot 0^2+0\right)$$

$$= \frac{8}{12}+1+2$$

$$= \underline{\underline{\frac{11}{3}}}$$

Käyrän ja y-akselin rajaama alue

esim 3 Määritä funktioiden $y = x^2, (x \geq 0)$, $y = 1$, $y = 4$ ja $x = 0$ rajaaman alueen ala.

Ratk. Pinta-alkion ala on $x\,dy = dA$

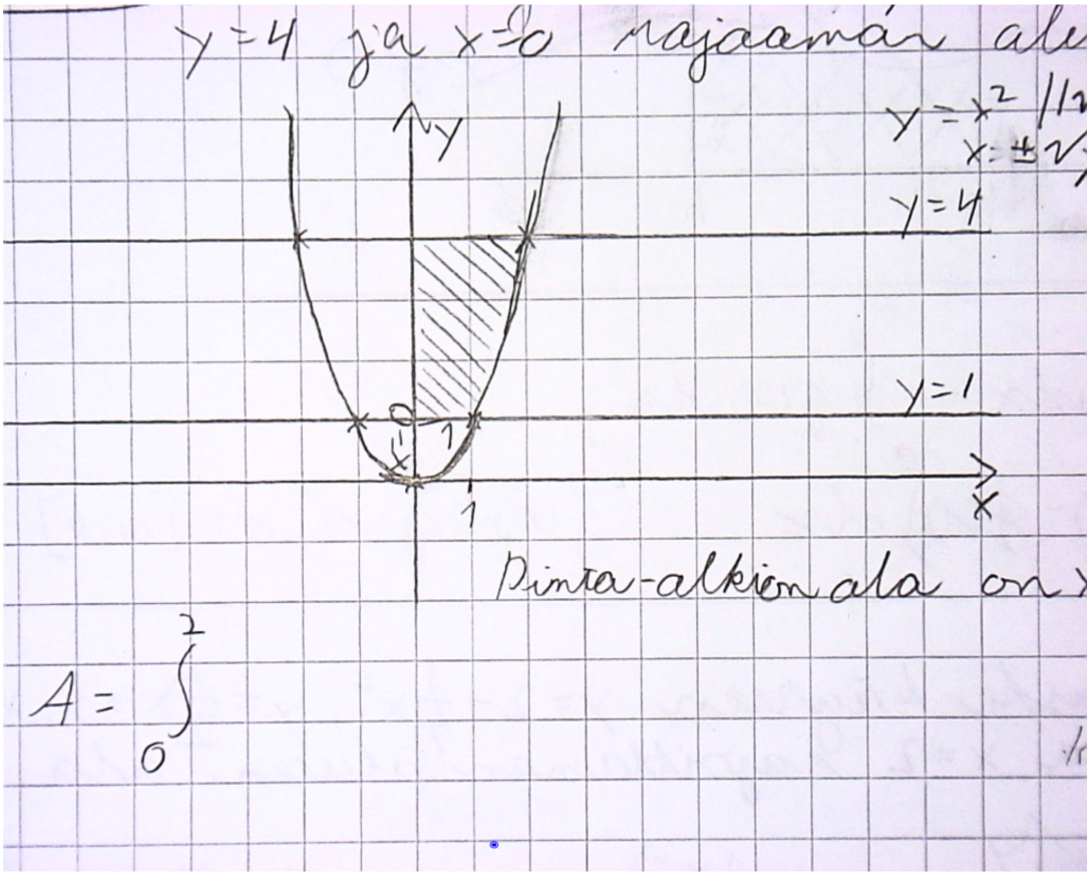
$$y = x^2 \quad | \sqrt{\ }$$

$$x = (\pm)\sqrt{y}$$

$$A = \int_1^4 dA = \int_1^4 x\,dy = \int_1^4 \sqrt{y}\,dy = \int_1^4 y^{\frac{1}{2}}\,dy$$

$$=$$

$$= 4\frac{2}{3}$$


y=4 ja x=0 rajaamán ale
y = x² ||
x = ±√
y
y=4
y=1
1
1
x
Pinta-alkion ala on x
A = ∫ 2 0