

III

$$\begin{aligned} \text{a) } \lim_{x \rightarrow \infty} \frac{\sqrt{9x^2+x}}{5x} &= \lim_{x \rightarrow \infty} \frac{\sqrt{x^2(9+\frac{1}{x})}}{5x} = \lim_{x \rightarrow \infty} \frac{\cancel{x} \sqrt{9+\frac{1}{x}}}{5\cancel{x}} \rightarrow 0 \\ &= \frac{\sqrt{9}}{5} = \frac{3}{5} \end{aligned}$$

$|x| = x, \text{ für } x \geq 0$

$$\begin{aligned} \text{b) } \lim_{x \rightarrow \infty} (\sqrt{x^2+3x} - x) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+3x}+x)(\sqrt{x^2+3x}-x)}{\sqrt{x^2+3x}+x} \\ &= \lim_{x \rightarrow \infty} \frac{x^2+3x-x^2}{\sqrt{x^2+3x}+x} \leftarrow \begin{aligned} &(a+b)(a-b) \\ &= a^2-b^2 \end{aligned} \\ &= \lim_{x \rightarrow \infty} \frac{3x}{\sqrt{x^2(1+\frac{3}{x})}+x} = \lim_{x \rightarrow \infty} \frac{3\cancel{x}}{\cancel{x} \sqrt{1+\frac{3}{x}} + \cancel{x} 1} \\ &= \lim_{x \rightarrow \infty} \frac{3}{\sqrt{1+\frac{3}{x}}+1} = \frac{3}{\sqrt{1+0}+1} = \frac{3}{2} \end{aligned}$$

$\frac{3}{x} \rightarrow 0$
 $\sqrt{1} = 1$

