t. 153, s. 32 Kerrotaan sulut auki ja muutetaan lausekkeet potenssimuotoon:

a) $$\int \left(1+\frac{1}{x}\right)^2 dx = \int \left(1+2\cdot\frac{1}{x}+\frac{1}{x^2}\right)dx = \int (1+2x^{-1}+x^{-2})dx$$

$$= x + 2\ln x + \frac{1}{-2+1}x^{-2+1} + C \; = x + 2\ln x - \frac{1}{x} + C$$

$$(a+b)^2 = a^2+2ab+b^2$$
$$(a-b)^2 = a^2-2ab+b^2$$

$$\frac{1}{x^n} = x^{-n}$$

b) $$\int \left(x-\frac{1}{x}\right)^2 dx = \int \left(x^2-2+\frac{1}{x^2}\right)dx = \int (x^2-2+x^{-2})dx$$

$$= \frac{1}{3}x^3 - 2x - x^{-1} + C \; = \frac{1}{3}x^3 - 2x - \frac{1}{x} + C$$

c) $$\int (\sqrt{x}+1)^2 dx = \int (x+2\sqrt{x}+1)dx = \int (x+2x^{\frac{1}{2}}+1)dx$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$= \frac{1}{2}x^2 + 2\cdot\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + x + C \; = \frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} + x + C$$

Lyhyemmin: $2\cdot\frac{2}{3}x^{\frac{3}{2}}$

$$= \frac{1}{2}x^2 + \frac{4}{3}x\sqrt{x} + x + C$$