

Sinin ja kosinin derivaatat

Tutkitaan sinin derivaattafunktiota.

Lause.

$$D \sin x = \cos x \quad \text{ja} \quad D \cos x = -\sin x.$$

Esimerkki 1. Derivoi a) $3 \sin x$ b) $\cos x - 5$ c) $\sin x - \cos x$

$$\text{a) } D 3 \sin x = 3 \cos x$$

$$\text{b) } D(\cos x - 5) = -\sin x + 0 = -\sin x$$

$$\text{c) } D(\sin x - \cos x) = \cos x - (-\sin x) = \cos x + \sin x$$

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Kerrataan tulon ja osamäärän derivaatan derivoimiskaavat.

$$D(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$D \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}, g(x) \neq 0$$

Esimerkki 2.

Derivoi a) $f(x) = x \cos x$ b) $g(x) = x^2 \sin x$

$$\text{c) } h(x) = \frac{\cos x}{\sin x}, x \neq n \cdot \pi$$

$$\text{a) } f'(x) = Dx \cdot \cos x + x \cdot D \cos x = \cos x - x \sin x$$

$$\text{b) } g'(x) = Dx^2 \cdot \sin x + x^2 \cdot D \sin x = 2x \sin x + x^2 \cos x$$

$$\begin{aligned} \text{c) } h'(x) &= \frac{D \cos x \cdot \sin x - \cos x \cdot D \sin x}{(\sin x)^2} = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} \\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}, x \neq n \cdot \pi \end{aligned}$$

$$\sin^2 x + \cos^2 x = 1$$