

t. 154, s. 31 Kerrotaan sulut auki ja muutetaan lausekkeet potenssimuotoon:

$$\begin{aligned} \text{a)} \quad \int \left(1 + \frac{1}{x}\right)^2 dx &= \int \left(1 + 2 \cdot \frac{1}{x} + \frac{1}{x^2}\right) dx = \int (1 + 2x^{-1} + x^{-2}) dx \\ &= x + 2 \ln x + \frac{1}{-2+1} x^{-2+1} + C = x + 2 \ln x - \frac{1}{x} + C \end{aligned}$$

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2 \end{aligned}$$

$$\frac{1}{x^n} = x^{-n}$$

$$\begin{aligned} \text{b)} \quad \int \left(x - \frac{1}{x}\right)^2 dx &= \int \left(x^2 - 2 + \frac{1}{x^2}\right) dx = \int (x^2 - 2 + x^{-2}) dx \\ &= \frac{1}{3} x^3 - 2x - x^{-1} + C = \frac{1}{3} x^3 - 2x - \frac{1}{x} + C \end{aligned}$$

$$\begin{aligned} \text{c)} \quad \int (\sqrt{x} + 1)^2 dx &= \int (x + 2\sqrt{x} + 1) dx = \int (x + 2x^{\frac{1}{2}} + 1) dx \\ &= \frac{1}{2} x^2 + 2 \cdot \frac{1}{\frac{1}{2} + 1} x^{\frac{1}{2} + 1} + x + C = \frac{1}{2} x^2 + \frac{4}{3} x^{\frac{3}{2}} + x + C \\ &= \frac{1}{2} x^2 + \frac{4}{3} x\sqrt{x} + x + C \end{aligned}$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

Lyhyemmin:
 $2 \cdot \frac{2}{3} x^{\frac{3}{2}}$