

Derivointiharjoitus rajankäynnin avulla

a) Funktion erotusosamäärälle $f: f(x) = 2x^2$ kohdassa $x = 3$ saadaan

$$\text{kun } \Delta x = 1: \quad \frac{2 \cdot (3+1)^2 - (2 \cdot 3^2)}{(3+1) - (3)} = \frac{32 - 18}{1} = \frac{14}{1} = 14$$

$$\text{kun } \Delta x = 0,5: \quad \frac{2 \cdot (3+0,5)^2 - (2 \cdot 3^2)}{(3+0,5) - (3)} = \frac{24,5 - 18}{0,5} = \frac{6,5}{0,5} = 13$$

$$\text{kun } \Delta x = 0,1: \quad \frac{2 \cdot (3+0,1)^2 - (2 \cdot 3^2)}{(3+0,1) - (3)} = \frac{19,22 - 18}{0,1} = \frac{1,22}{0,1} = 12,2$$

$$\text{kun } \Delta x = 0,01: \quad \frac{2 \cdot (3+0,01)^2 - (2 \cdot 3^2)}{(3+0,01) - (3)} = \frac{18,1202 - 18}{0,01} = \frac{0,1202}{0,01} = 12,02$$

$$\text{kun } \Delta x = 0,001: \quad \frac{2 \cdot (3+0,001)^2 - (2 \cdot 3^2)}{(3+0,001) - (3)} = \frac{18,012002 - 18}{0,001} = \frac{0,012002}{0,001} = 12,002$$

$$\text{kun } \Delta x = 0,0001: \quad \frac{2 \cdot (3+0,0001)^2 - (2 \cdot 3^2)}{(3+0,0001) - (3)} = \frac{18,00120002 - 18}{0,0001} = 12,0002$$

Raja-arvo näyttäisi olevan 12 kohdassa $x = 3$.

b) Funktion erotusosamäärälle $f: f(x) = x - x^2$ kohdassa $x = 5$ saadaan

$$\text{kun } \Delta x = 1: \quad \frac{[(5+1) - (5+1)^2] - [5 - 5^2]}{(5+1) - (5)} = \frac{-30 - (-20)}{1} = \frac{-10}{1} = -10$$

$$\text{kun } \Delta x = 0,5: \quad \frac{[\quad] - [\quad]}{(\quad) - (\quad)} = \frac{-24,75 + 20}{0,5} = \frac{-4,75}{0,5} = -9,5$$

$$\text{kun } \Delta x = 0,1: \quad \frac{[\quad] - [\quad]}{(\quad) - (\quad)} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,01: \quad \frac{[\quad] - [\quad]}{(\quad) - (\quad)} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,001: \quad \frac{[\quad] - [\quad]}{(\quad) - (\quad)} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,0001: \quad \frac{[\quad] - [\quad]}{(\quad) - (\quad)} = \underline{\hspace{2cm}} =$$

Raja-arvo näyttäisi olevan kohdassa $x = 5$.

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c) Funktion erotusosamäärälle $f: f(x) = x^2 - 4$ kohdassa $x = -1$ saadaan

$$\text{kun } \Delta x = 1: \frac{[(-1+1)^2-4]-[(-1)^2-4]}{(-1+1)-(-1)} = \frac{-4-(-3)}{1} = \frac{-1}{1} = -1$$

$$\text{kun } \Delta x = 0,5: \frac{[(-1+0,5)^2-4]-[-3]}{(-1+0,5)-(-1)} = \frac{-3,75+3}{0,5} = \frac{-0,75}{0,5} = -1,5$$

$$\text{kun } \Delta x = 0,1: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,01: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,001: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,0001: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} =$$

Raja-arvo näyttäisi olevan kohdassa $x = -1$.

d) Funktion erotusosamäärälle $f: f(x) = x - 3x^2$ kohdassa $x = 0$ saadaan

$$\text{kun } \Delta x = 1: \frac{[(0+1)-3\cdot(0+1)^2]-[0-3\cdot 0^2]}{(-1+1)-(-1)} = \frac{-2-(0)}{1} = \frac{-2}{1} = -2$$

$$\text{kun } \Delta x = 0,5: \frac{[(0+0,5)-3\cdot(0+0,5)^2]-0}{(-1+0,5)-(-1)} = \frac{-0,25-(0)}{0,5} = \frac{-0,25}{0,5} = -0,5$$

$$\text{kun } \Delta x = 0,1: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,01: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,001: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} =$$

$$\text{kun } \Delta x = 0,0001: \frac{[\quad]-[\quad]}{(\quad)-(\quad)} = \underline{\hspace{2cm}} =$$

Raja-arvo näyttäisi olevan kohdassa $x = 0$.