13.4A Määritä.

a) $\int x \sin x^2 \, dx$

b) $\int \frac{x^3}{(x^4+1)^2} dx$

c) $\int \frac{5x^3}{x^4+1} dx$

4. $\int f'(x)\, g'(f(x))\, dx = g(f(x)) + C$

$\int f'(x) g(f(x)) dx = G(f(x)) + C$, jossa $G(x) = \int g(x) dx$

a) $\int x \underbrace{\sin x^2}_{\text{yhdistetty funktio}} dx = \frac{1}{2} \cdot \int 2x \sin x^2 dx = \frac{1}{2} \cdot (-\cos(x^2)) + C = -\frac{1}{2}\cos(x^2) + C$

$g' = \sin x \quad g = \int \sin x = -\cos x$

$f = x^2 \quad f' = 2x$

$g' = x^{-2} \Rightarrow \int x^{-2} dx = \frac{1}{-1} x^{-1} = -\frac{1}{x}$

$g'(f(x))$

9. $\int f'(x)(f(x))^n dx = \frac{(f(x))^{n+1}}{n+1} + C \quad (n \neq -1)$

10. $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C \quad (f(x) \neq 0)$

b) $= \int x^3 (x^4+1)^{-2} dx = \frac{1}{4} \int 4x^3 (x^4+1)^{-2} dx = \frac{1}{4} \cdot \left(-\frac{1}{x^4+1}\right) + C = -\frac{1}{4(x^4+1)} + C$

c) $= 3 \cdot \frac{1}{4} \int \frac{4x^3}{x^4+1} dx = \frac{3}{4} \ln|\underbrace{x^4+1}_{>0}| + C = \frac{3}{4} \ln(x^4+1) + C$

13.9A **1.** Osoita, että yhtälö $\dfrac{4}{4-x^2} = \dfrac{1}{2+x} + \dfrac{1}{2-x}$ on voimassa kaikilla $x \neq \pm 2$.

2. Laske integraali $\displaystyle\int_{-1}^{1} \frac{1}{2+x}\,\mathrm{d}x$.

3. Laske integraali $\displaystyle\int_{-1}^{1} \frac{1}{4-x^2}\,\mathrm{d}x$.

(yo pitkä k2021/4)

$= \frac{1}{4}\int_{-1}^{1} \frac{4}{4-x^2}\,dx = \frac{1}{4}\int_{-1}^{1} \frac{1}{2+x} + \frac{1}{2-x}\,dx =$

kohta 1.

$\frac{1}{4}\int_{-1}^{1} \frac{1}{2+x}\,dx + \frac{1}{4}\int_{-1}^{1} \frac{1}{2-x}\,dx = \frac{1}{4}\Big[\ln|2+x| - \ln|2-x|\Big]_{-1}^{1} =$

$g(x) \to g'(x) = (-1)$

$\frac{1}{4}(\ln 3 - \underbrace{\ln 1}_{=0}) - \frac{1}{4}(\underbrace{\ln 1}_{=0} - \ln 3) = \frac{1}{4}(\ln 3 + \ln 3) = \frac{1}{4}\cdot 2\ln 3 = \frac{1}{2}\ln 3 = \ln 3^{\frac{1}{2}} = \ln\sqrt{3}$

$\int_{-1}^{1} \frac{1}{4-x^2}\,dx$

$\frac{\ln(3)}{2}$