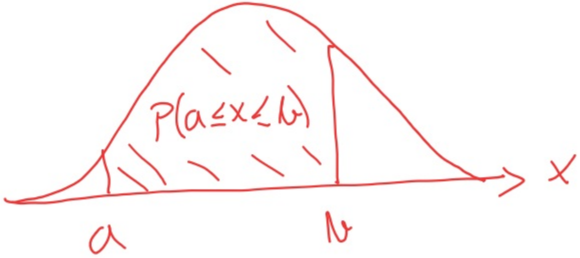
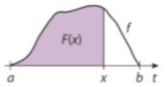
Jatkuvan satunnaismuuttujan X tiheysfunktio $f(x)$

1° $f(x) \geq 0$, kun $x \in \mathbb{R}$

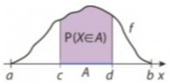
2° $\int_{-\infty}^{\infty} f(x)\,dx = 1$

3° $P(a \leq X \leq b) = \int_a^b f(x)\,dx$



(1) $\forall x \in E$: $f(x) \geq 0$	
(2) $\int_a^b f(x)\mathrm{d}x = 1$ (jos $E = \mathbb{R}$, niin ehto on $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$)	

Muuttujaan liittyvä *kertymäfunktio* on $F(x) = \mathrm{P}(X \leq x) = \int_a^x f(t)\mathrm{d}t$.

todennäköisyys	$\mathrm{P}(X \in A) = \int_c^d f(x)\mathrm{d}x = F(d) - F(c)$	
odotusarvo	$\mathrm{E}(X) = \mu = \int_a^b x f(x)\mathrm{d}x$	
keskihajonta	$\mathrm{D}(X) = \sigma = \sqrt{\int_a^b (x-\mu)^2 f(x)\mathrm{d}x} = \sqrt{\int_a^b x^2 f(x)\mathrm{d}x - \mu^2}$	
varianssi	σ^2	

12.4 Satunnaismuuttujan X tiheysfunktio on

$$f(x) = \begin{cases} 0, & \text{kun } x \leq 0 \\ \frac{a}{\sqrt[4]{x}}, & \text{kun } 0 < x \leq 1 \\ 0, & \text{kun } x > 1. \end{cases}$$

Määritä

a) vakion a arvo

b) satunnaismuuttujan X odotusarvo

c) satunnaismuuttujan X keskihajonta.

a) $f(x)$ pitää olla ≥ 0 (ehto 1°)

$\frac{a}{\sqrt[4]{x}} \geq 0$, kun $0 < x \leq 1$

$\sqrt[4]{x} > 0 \rightarrow a \geq 0$

(ehto 2°) $\int_0^1 f(x)\,dx = 1$

Lasketaan ensin

$$\int_t^1 a x^{-\frac{1}{4}}\,dx = a \Big/_t^1 \frac{4}{3} x^{\frac{3}{4}} = a\left(\frac{4}{3} \cdot 1^{\frac{3}{4}} - \frac{4}{3} t^{\frac{3}{4}}\right)$$

Kun $t \to 0$: $\lim_{t \to 0} a\left(\frac{4}{3} - \underbrace{\frac{4}{3} t^{\frac{3}{4}}}_{\to 0}\right) = \frac{4}{3}a$

$\frac{4}{3}a = 1 \Leftrightarrow \underline{\underline{a = \frac{3}{4}}}$

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$\text{solve}\left(\int_0^1 \frac{a}{4\sqrt{x}}\,dx = 1, a\right)$ $\quad \left\{a = \frac{3}{4}\right\}$

b) $E(x) = \int_0^1 x \cdot \frac{3}{4\sqrt[4]{x}}\,dx$

$\int_0^1 x * \frac{3}{4*4\sqrt{x}}\,dx$ $\quad \frac{3}{7}$

c) $\delta(x) = \sqrt{\int_0^1 \left(x - \frac{3}{7}\right)^2 \cdot \frac{3}{4\sqrt[4]{x}}\,dx}$

$\sqrt{\int_0^1 \left(x - \frac{3}{7}\right)^2 * \frac{3}{4*4\sqrt{x}}\,dx}$ $\quad \frac{4 \cdot \sqrt{33}}{77}$

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$\sqrt{\int_0^1 x^2 * \frac{3}{4*4\sqrt{x}}\,dx - \left(\frac{3}{7}\right)^2}$ $\quad \frac{4 \cdot \sqrt{33}}{77}$