6.19 ★★★ Määritä funktion $f(x)=\int_0^x (t\sqrt{x}-\frac{1}{\sqrt{x}})\,dt$ nollakohdat.

$=$ Mj. $x>0$

$$= \Big/_0^x \frac{1}{2}t^2\cdot\sqrt{x} - \frac{1}{\sqrt{x}}\cdot t =$$

$$\frac{1}{2}x^2\sqrt{x} - \frac{1}{\sqrt{x}}\cdot x - \left(\frac{1}{2}\cdot 0^2\cdot\sqrt{x} - \frac{1}{\sqrt{x}}\cdot 0\right) =$$

$$\frac{1}{2}x^2\sqrt{x} - \sqrt{x}$$

nollakohdat: $\sqrt{x}\left(\frac{1}{2}x^2-1\right)=0$

$\sqrt{x}=0 \quad \vee \quad \frac{1}{2}x^2-1=0$

$(\underline{\underline{x=0}})$ $\quad \frac{1}{2}x^2=1 \quad \| \cdot 2$

$x^2=2 \quad \| \sqrt{\ }$

$\underline{\underline{x=\pm\sqrt{2}}}$

V: $\underline{\underline{x=\sqrt{2}}}$

yhdistetty funktio:

Derivointi: $(g(f(x)))' = g'(f(x)) \cdot f'(x)$

Esim. $D\,(x^2+3)^4 = 4(x^2+3)^3 \cdot 2x = \underline{8x(x^2+3)^3}$

$g(x) = x^4$, $f(x) = x^2+3$

$g'(x) = 4x^3$, $f'(x) = 2x$

4. $\int f'(x)\, g'(f(x))\, \mathrm{d}x = g(f(x)) + C$

Integrointi: $\int f'(x)\, g(f(x))\, dx = G(f(x)) + C$, kun $G'(x) = g(x)$

Esim. $\int 8x(x^2+3)^3\, dx = \int 4 \cdot \underline{2x}\,(x^2+3)^3\, dx = 4 \cdot \frac{1}{4}(x^2+3)^4 + C$

vakio

$g(x) = x^3$, $f(x) = x^2+3$

$G(x) = \frac{1}{4}x^4$, $f'(x) = 2x$

$= \underline{(x^2+3)^4 + C}$

Esim. $\int x^2\sqrt{x^3+2}\,dx$

$$3\cdot\frac{1}{3}=1$$

$$= \int x^2(x^3+2)^{\frac{1}{2}}\,dx = \frac{1}{3}\cdot\int \underline{3\cdot x^2}(x^3+2)^{\frac{1}{2}}\,dx =$$

$$g(x)=x^{\frac{1}{2}}, \quad f(x)=x^3+2$$

$$G(x)=\frac{2}{3}x^{\frac{3}{2}}, \quad f'(x)=\underline{3x^2}$$

$$\frac{1}{3}\cdot\frac{2}{3}(x^3+2)^{\frac{3}{2}}+C=$$

$$\frac{2}{9}(x^3+2)^{\frac{3}{2}}+C=$$

$$\underline{\underline{\frac{2}{9}(x^3+2)\sqrt{x^3+2}+C}}$$

$$\int \frac{x}{\sqrt{x^2+5}}\,dx = \int x\,(x^2+5)^{-\frac{1}{2}}\,dx = \frac{1}{2}\int 2\cdot x(x^2+5)^{-\frac{1}{2}}\,dx$$

$g(x) = x^{-\frac{1}{2}}$ $\quad$ $f(x) = x^2+5$

$G(x) = 2\,x^{\frac{1}{2}}$ $\quad$ $f'(x) = 2x$

$$= \frac{1}{2}\,\underbrace{2(x^2+5)^{\frac{1}{2}}}\,+ C$$

$$= (x^2+5)^{\frac{1}{2}} + C$$

$$= \sqrt{x^2+5} + C$$