4.21 Olkoon $f''(x) = 3x - 5$. Määritä se funktio f, jonka kuvaajan pisteeseen $(1, 1)$ piirretyn tangentin kulmakerroin on -2.

Tangentin kulmakerroin $k = f'(x_0)$, jossa x_0 on kuvaajan pisteen x-koordinaatti $\Rightarrow f'(1) = -2$

$$f'(x) = \int f''(x)\,dx = \int 3x - 5\,dx$$

$$f'(x) = 3 \cdot \frac{1}{2}x^2 - 5x + C$$

$$\Rightarrow \frac{3}{2} \cdot 1^2 - 5 \cdot 1 + C = -2$$

$$C = \frac{10}{3} - 2 = \frac{4}{3}$$

$$f(x) = \int f'(x)\,dx = \int \left(\frac{3}{2}x^2 - 5x + \frac{3}{4}\right)dx =$$

$$\frac{3}{2} \cdot \frac{1}{3}x^3 - 5 \cdot \frac{1}{2}x^2 + \frac{3}{4}x + D = \frac{1}{2}x^3 - \frac{5}{2}x^2 + \frac{3}{4}x + D$$

Kuvaaja kulkee pisteen $(1, 1)$ kautta $\Rightarrow f(1) = 1$

$$\frac{1}{2} \cdot 1^3 - \frac{5}{2} \cdot 1^2 + \frac{3}{4} \cdot 1 + D = 1$$

$$D = \frac{9}{4}$$

Vast: $f(x) = \frac{1}{2}x^3 - \frac{5}{2}x^2 + \frac{3}{4}x + \frac{9}{4}$

Määrätty integraali

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

esim: $$\int_{-2}^{1} (2x-3)\,dx = \Big/_{-2}^{1} (x^2-3x) = 1^2 - 3\cdot 1 - \left((-2)^2 - 3\cdot(-2)\right)$$

$$= 1 - 3 - (4+6) = \underline{\underline{-12}}$$

5.4 Laske.

~~CAS~~

a) $\int_0^3 (x(x-2))\,dx$ **b)** $\int_{-1}^2 (x+1)^2\,dx$

b) $\int_{-1}^{2} (x^2+2x+1)\,dx = \Big/_{-1}^{2} \left(\frac{1}{3}x^3 + x^2 + x\right)$

$= \frac{1}{3} \cdot 2^3 + 2^2 + 2 - \left(\frac{1}{3} \cdot (-1)^3 + (-1)^2 - 1\right)$

$= \frac{8}{3} + 4 + 2 - \left(-\frac{1}{3} + 1 - 1\right)$

$= \frac{8}{3} + 4 + 2 + \frac{1}{3} - 1 + 1 = \underline{9}$

5.8 Määritä funktion $f(x)=\int_2^x (2t^3-3t^2)\,dt$ nollakohdat.

$$\Big/_2^x \left(2\cdot\frac{1}{4}t^4 - t^3\right) = \Big/_2^x \left(\frac{1}{2}t^4 - t^3\right)$$

$$= \frac{1}{2}x^4 - x^3 - \left(\frac{1}{2}\cdot 2^4 - 2^3\right)$$

$$= \frac{1}{2}x^4 - x^3 - (8-8) = \frac{1}{2}x^4 - x^3 = f(x)$$

$$\int_2^x (2t^3-3t^2)\,dt \qquad \frac{x^4}{2}-x^3$$

Nollakohdat: $\frac{1}{2}x^4 - x^3 = 0$

$$x^3\left(\frac{1}{2}x - 1\right) = 0$$

$$x^3 = 0 \lor \frac{1}{2}x - 1 = 0$$

$$\underline{\underline{x=0}} \qquad \underline{\underline{x=2}}$$