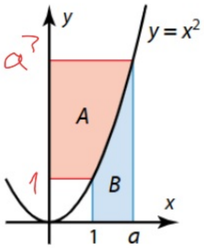
12.18 Osoita, että kuvassa alueen A pinta-ala on kaksinkertainen alueen B pinta-alaan verrattuna aina, kun $a > 1$.



Ratkaistaan x:

$y = x^2 \quad || \sqrt{\ }$

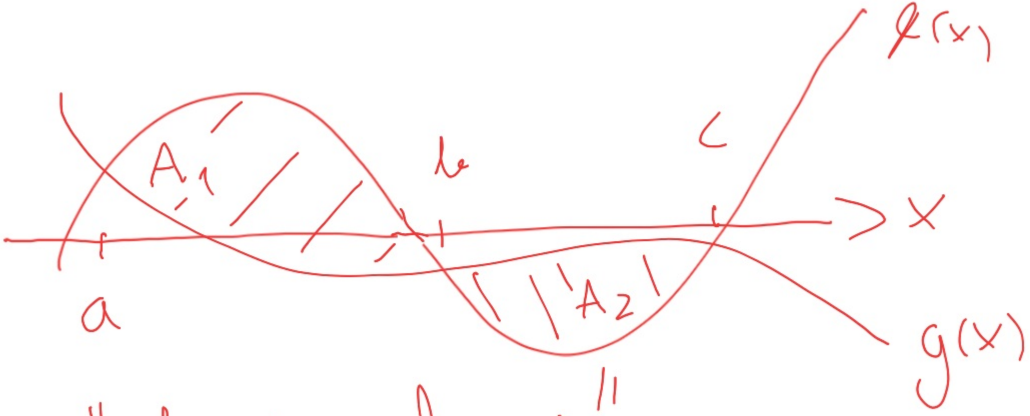
$x = (\pm)\sqrt{y}$

$$B = \int_1^a x^2\,dx = \Big/_1^a \frac{1}{3}x^3 = \frac{1}{3}a^3 - \frac{1}{3}$$

$$A = \int_1^{a^2} \sqrt{y}\,dy = \int_1^{a^2} y^{\frac{1}{2}}\,dy = \Big/_1^{a^2} \frac{2}{3}y^{\frac{3}{2}} = \frac{2}{3}\Big/_1^{a^2} y\sqrt{y} =$$

$$\frac{2}{3}\left(a^2\sqrt{a^2} - 1\cdot\sqrt{1}\right) = \frac{2}{3}(a^3 - 1) = 2B \; \square$$

Kahden käyrän rajaama pinta-ala



$$A_1 = \int_a^b \left(f(x) - g(x)\right) dx$$

$$A_2 = \int_b^c \left(g(x) - f(x)\right) dx$$

"ylempi – alempi"

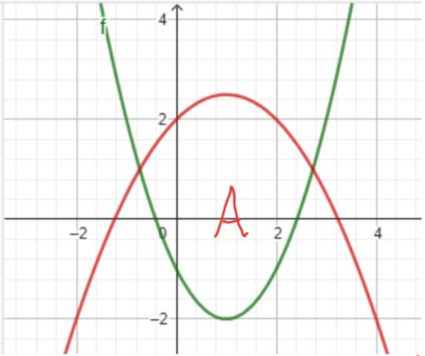
$f(x) = x^2 - 2x - 1$

$g(x) = -\frac{1}{2}x^2 + x + 2$

$h(x) = x^3$

$p(x) = x^4 - 2x^2$

Syöttökenttä...



Kuvaajien rajoittama pinta-ala

Leikkauspisteet

$f(x) = g(x)$

g(x) on ylempi kuvaaja

$$\text{solve}\left(x^2-2x-1=-\frac{1}{2}x^2+x+2\right)$$

$$\{x=-\sqrt{3}+1,\ x=\sqrt{3}+1\}$$

$$\int_{-\sqrt{3}+1}^{\sqrt{3}+1} -\frac{3}{2}x^2+3x+3dx$$

$$\frac{-(\sqrt{3}+1)^3}{2}-\frac{(\sqrt{3}-1)^3}{2}+\frac{3\cdot(\sqrt{3}+1)^2}{2}-\frac{3\cdot(\sqrt{3}-1)^2}{2}+3\cdot(\sqrt{3}+1)+3\cdot(\sqrt{3}-1)$$

$$\int_{-\sqrt{3}+1}^{\sqrt{3}+1} -\frac{3}{2}x^2+3x+3dx$$

$$10.39230485$$

$$A=\int_{-\sqrt{3}+1}^{\sqrt{3}+1}\left(-\frac{1}{2}x^2+x+2-(x^2-2x-1)\right)dx=\int_{-\sqrt{3}+1}^{\sqrt{3}+1}\left(-\frac{3}{2}x^2+3x+3\right)dx$$

$f(x) = x^2 - 2x - 1$

$g(x) = -\frac{1}{2}x^2 + x + 2$

$h(x) = x^3$

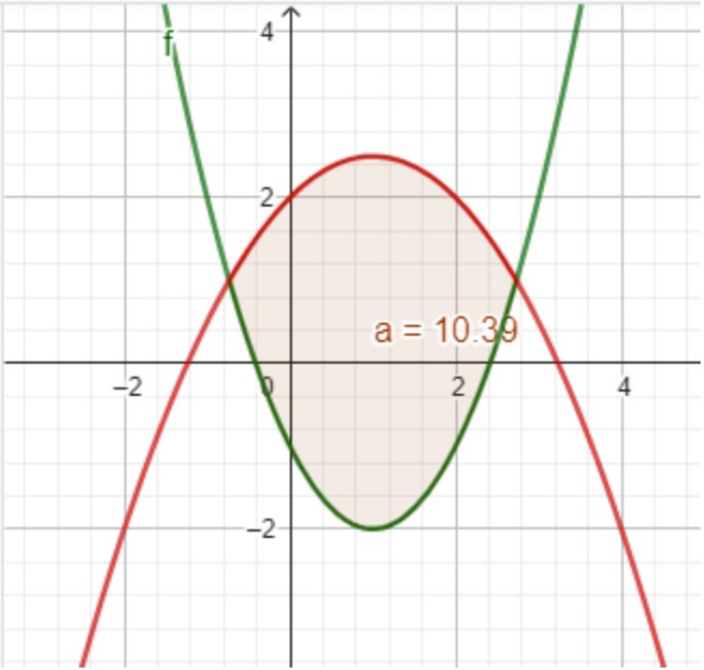
$p(x) = x^4 - 2x^2$

alaraja yläraja

$a = \text{IntegraaliVäli}(g, f, -\sqrt{3} + 1, \sqrt{3} + 1)$

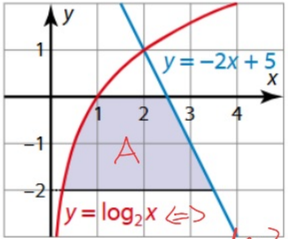
$= 10.39$

ylempi alempi



13.5 Laske väritetyn alueen pinta-ala.

E3



$\Leftrightarrow \; -2x = y - 5$

$x = -\frac{1}{2}y + \frac{5}{2}$

$\Leftrightarrow \; x = 2^y = e^{\ln 2\, y}$

$$A = \int_{-2}^{0}\left(-\frac{1}{2}y + \frac{5}{2} - e^{\ln 2\, y}\right)dy = \Big/_{-2}^{0} -\frac{1}{4}y^2 + \frac{5}{2}y - \overbrace{\frac{1}{\ln 2}e^{\ln 2\, y}}^{\frac{1}{\ln 2}2^y} =$$

$$-\frac{1}{4}\cdot 0 + \frac{5}{2}\cdot 0 - \frac{1}{\ln 2}\cdot 2^0 - \left(-\frac{1}{4}\cdot(-2)^2 + \frac{5}{2}\cdot(-2) - \frac{1}{\ln 2}\cdot 2^{-2}\right) =$$

$$-\frac{1}{\ln 2} - \left(-1 - 5 - \frac{1}{\ln 2}\cdot\frac{1}{4}\right) =$$

$$\underline{\underline{6 - \frac{3}{4\ln 2}}}$$

$$\int_{-2}^{0} -\frac{1}{2}y + \frac{5}{2} - 2^y \, dy$$

$$\frac{-3}{4\cdot\ln(2)} + 6$$