Trigonometriset funktiot

Derivaatta: $D \sin x = \cos x$ | $D \sin(f(x)) = f'(x) \cos(f(x))$

$D \cos x = -\sin x$ | $D \cos(f(x)) = -f'(x) \sin(f(x))$

Integraali: $\int \cos x \, dx = \sin x + C$ | $\int f'(x) \cos(f(x)) \, dx = \sin(f(x)) + C$

$\int \sin x \, dx = -\cos x + C$ | $\int f'(x) \sin(f(x)) \, dx = -\cos(f(x)) + C$

Örn. $\int \cos(3x)\,dx = \frac{1}{3}\int 3\cos(3x)\,dx = \frac{1}{3}\sin(3x) + c$

$g(x) = \cos x \quad f(x) = 3x$

$G(x) = \sin x \quad f'(x) = 3$

Örn. $\int \frac{1}{2}x^2 \sin(x^3+4)\,dx = \frac{1}{2}\cdot\frac{1}{3}\int 3x^2 \sin(x^3+4)\,dx =$

$-\frac{1}{6}\cos(x^3+4) + c$

$g(x) = \sin x \quad f(x) = x^3+4$

$f'(x) = 3x^2$

$G(x) = -\cos x$

9.7 Määritä funktion $f(x) = 2\sin x + \cos 3x + e^{-x}$ se integraalifunktio F, jolla $F(0) = 6$.

$F(x) = \int (2\sin x + \cos(3x) + e^{-x})\,dx$

$= -2\cos x + \frac{1}{3}\sin(3x) - e^{-x} + C$

$F(0) = 6$

$-2\underbrace{\cos 0}_{=1} + \frac{1}{3}\underbrace{\sin(3\cdot 0)}_{=0} - \underbrace{e^{-0}}_{=1} + C = 6$

$-2 + 0 - 1 + C = 6$

$C = 9$

Vast: $-2\cos x + \frac{1}{3}\sin(3x) - e^{-x} + 9$

9.17 Integroi, kun $0 < x < \frac{\pi}{2}$.

a) $\frac{\sin x}{\cos^2 x}$ **b)** $\sin x\sqrt{\cos x}$

b) $= \int \sin x\,(\cos x)^{\frac{1}{2}}\,dx =$

$g(x) = x^{\frac{1}{2}}$, $f(x) = \cos x$

$G(x) = \frac{2}{3}x^{\frac{3}{2}}$, $f'(x) = -\sin x$

$-\int -\sin x\,(\cos x)^{\frac{1}{2}}\,dx =$

$-\frac{2}{3}(\cos x)^{\frac{3}{2}} + C =$

$-\frac{2}{3}(\cos x)\sqrt{\cos x} + C$

9.18 Integroi.

CAS E4

a) $\cos^2 x$

b) $\cos^2 x - \sin^2 x$

c) $(\cos x - \sin x)^2$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$2\cos^2 x = \cos(2x) + 1 \quad || :2$

$\cos^2 x = \frac{1}{2}\cos(2x) + \frac{1}{2}$

a) $\int (\cos x)^2 \, dx = \int \left(\frac{1}{2}\cos(2x) + \frac{1}{2}\right) dx$

$= \frac{1}{2} \cdot \frac{1}{2} \int 2\cos(2x) \, dx + \int \frac{1}{2} \, dx$

$= \frac{1}{4}\sin(2x) + \frac{1}{2}x + C$

Osamääräntä

Derivaatta: $D\ln|x| = \frac{1}{x}$, $x \neq 0$ | $D\ln|f(x)| = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$, $f(x) \neq 0$

Integraali: $\int \frac{1}{x}\,dx = \ln|x| + C$ | $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$

Esim. $\int -\frac{5}{x}\,dx = \int -5 \cdot \frac{1}{x}\,dx = -5\ln|x| + C$

Örn. $\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx = \frac{1}{2}\ln|x^2+4| + c = \frac{1}{2}\ln(x^2+4)+c$

$\underbrace{x^2+4}_{f(x)} \Rightarrow f'(x) = 2x$

$x^2+4>0$

$|x^2+4| = x^2+4$

Örn. $\int \frac{x^2}{x^3-1}\,dx$, iken $x<1$

$= \frac{1}{3}\int \frac{3x^2}{x^3-1}\,dx = \frac{1}{3}\ln|x^3-1| + c = \frac{1}{3}\ln(1-x^3)+c$

$\frac{f'}{f}$

$x^3-1<0$, iken $x<1$

$|x^3-1| = -x^3+1 = 1-x^3$

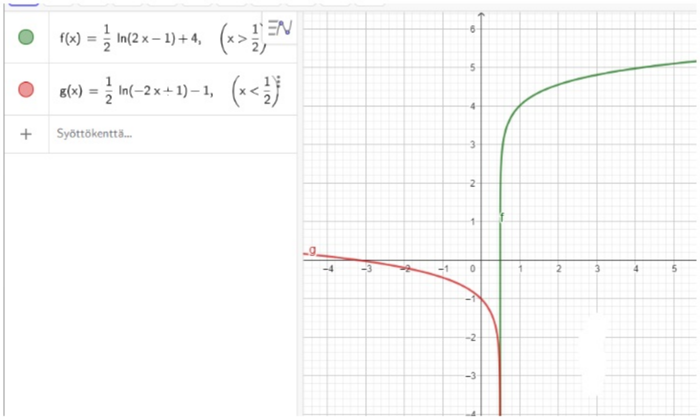
10.5 Määritä funktion $f(x) = \dfrac{1}{2x-1}$ se integraalifunktio F, jonka kuvaaja kulkee pisteiden $(0, -1)$ ja $(1, 4)$ kautta. Piirrä funktion F kuvaaja.

E3

$2x - 1 = 0$, kun $x = \frac{1}{2}$

$$\frac{1}{2}\int \frac{2}{2x-1}\,dx = \frac{1}{2}\ln|2x-1|$$

	$\frac{1}{2}$	
$2x-1$	−	+
	$\lvert 2x-1\rvert = -2x+1$	$\lvert 2x-1\rvert = 2x-1$



Define $F(x)=\begin{cases}\frac{1}{2}*\ln(2x-1)+C, & x>\frac{1}{2}\\ \frac{1}{2}*\ln(-2x+1)+D, & x<\frac{1}{2}\end{cases}$

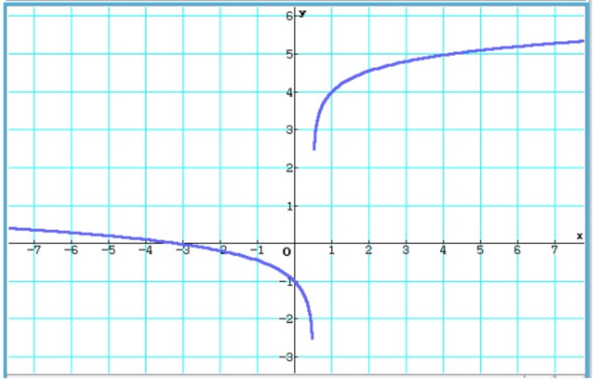
done

$\begin{cases}F(0)=-1\\ F(1)=4\end{cases}\Big|_{C,D}$

{C=4, D=−1}

Define $F(x)=\begin{cases}\frac{1}{2}*\ln(2x-1)+4, & x>\frac{1}{2}\\ \frac{1}{2}*\ln(-2x+1)-1, & x<\frac{1}{2}\end{cases}$

done



10.10 Määritä $\int_0^{\frac{\pi}{4}} \frac{2\sin x}{5\cos x}\,dx.$ =

CAS

$$\frac{2}{5}\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx = -\frac{2}{5}\int_0^{\frac{\pi}{4}} \frac{-\sin x}{\cos x}\,dx = -\frac{2}{5}\Big/_0^{\frac{\pi}{4}} \ln|\cos x| =$$

$$-\frac{2}{5}\left(\ln\left|\cos\frac{\pi}{4}\right| - \ln|\cos 0|\right) = -\frac{2}{5}\left(\ln\frac{1}{\sqrt{2}} - \underbrace{\ln 1}_{=0}\right) = -\frac{2}{5}\ln\frac{1}{\sqrt{2}} =$$

$$-\frac{2}{5}\ln 2^{-\frac{1}{2}} = -\frac{2}{5}\cdot\left(-\frac{1}{2}\right)\ln 2 = \underline{\underline{\frac{1}{5}\ln 2}}$$