7.16 Määritä funktion $f(x)=(2x-1)(x^2-x)^2$ se integraalifunktio F, jonka pienin arvo on -1.

$f(x) = F'(x)$

$F(x)$:n pienin arvo löytyy $F'(x)$:n nollakohdista. | Kulkukaavio:

$f(x)=0$, kun $(2x-1)(x^2-x)^2=0$

$2x-1=0 \lor x^2-x=0$

$2x=1 \qquad x(x-1)=0$

$x=\frac{1}{2} \qquad x=0 \lor x-1=0$

$x=1$

		0		$\frac{1}{2}$		1	
$f(x)$	$-$		$-$		$+$		$+$
$F(x)$	$\searrow$		$\searrow$		$\nearrow$		$\nearrow$

min kohta $\Rightarrow F\left(\frac{1}{2}\right)=-1$

$$\int (2x-1)(x^2-x)^2\,dx = \frac{1}{3}(x^2-x)^3 + C$$

$g = x^2$, $\quad g(x) = x^2 - x$

$G = \frac{1}{3}x^3$, $\quad f'(x) = 2x-1$

$$\frac{1}{3}\left(\frac{1}{2}^2-\frac{1}{2}\right)^3 + C = -1$$

$$\frac{1}{3}\left(-\frac{1}{4}\right)^3 + C = -1$$

$$C = -\frac{191}{192}$$

$$F(x) = \frac{1}{3}(x^2-x)^3 - \frac{191}{192}$$

Eksponenttifunktio e^x

Derivointi: $De^x = e^x$ $\qquad De^{f(x)} = e^{f(x)} \cdot f'(x)$

Integrointi: $\boxed{\int e^x \, dx = e^x + c}$

Esim. $\int e^{2x} \, dx = \underbrace{\frac{1}{2} \int 2}_{\frac{1}{2} \cdot 2 = 1} \cdot e^{2x} \, dx = \frac{1}{2} e^{2x} + c$

$g(x) = e^x \qquad f(x) = 2x$

$G(x) = e^x \qquad f'(x) = 2$

Esim. $\int (x^2 e^{-x^3+2})\,dx = -\frac{1}{3}\int \underline{-3}\cdot x^2 e^{-x^3+2}\,dx = -\frac{1}{3}\cdot e^{-x^3+2} + C$

$g(x) = e^x$, $f(x) = -x^3+2$

$G(x) = e^x$, $f'(x) = \underline{-3}x^2$

8.6 Integroi.

a) $\frac{3}{e^x}$

b) $\frac{x}{e^{x^2}}$

c) $\frac{4x^2}{e^{x^3}}$

c) $\frac{4x^2}{e^{x^3}} = 4x^2\cdot e^{-x^3}$

$$\int 4x^2 e^{-x^3}\,dx = -\frac{1}{3}\int 4(-3)x^2 e^{-x^3}\,dx = -\frac{4}{3}e^{-x^3} + C$$

$f(x) = -x^3$

$f'(x) = -3x^2$