

Yhdistettyjen funktioiden integrointi.

Derivoimalla: $(g(f(x)))' = g'(f(x)) \cdot f'(x)$

Esim. $D \quad (x^2+3)^4 = 4(x^2+3)^3 \cdot 2x = \underline{\underline{8x(x^2+3)^3}}$

$g(x) = x^4$, $f(x) = x^2+3$
 $g'(x) = 4x^3$, $f'(x) = 2x$

MAOL

$4. \int f'(x) g'(f(x)) dx = g(f(x)) + C$

Integrointi: $\int f'(x) \cdot g(f(x)) dx = G(f(x)) + C$ kun $G(x) = g(x)$

Esim. $\int 8x(x^2+3)^3 dx = \int 4 \cdot 2x(x^2+3)^3 dx = 4 \cdot \underbrace{\frac{1}{4}(x^2+3)^4}_{G(f(x))} + C$
 $G(x) = \frac{1}{4}x^4$, $f(x) = x^2+3$
 $G'(x) = x^3$, $f'(x) = 2x$

$= \underline{\underline{(x^2+3)^4 + C}}$

Exm. $\int (6x^2 \sqrt{x^3+2}) dx =$
 $\int (6x^2 (x^3+2)^{\frac{1}{2}}) dx = \int \underbrace{2 \cdot 3x^2}_{f'(x)} (x^3+2)^{\frac{1}{2}} dx = \underbrace{\frac{2}{3} (x^3+2)^{\frac{3}{2}}}_{G(f(x))} + C$

$$g(x) = x^{\frac{1}{2}}$$

$$G(x) = \frac{2}{3} x^{\frac{3}{2}}$$

$$f(x) = x^3 + 2$$

$$f'(x) = 3x^2$$

$$= \frac{4}{3} (x^3+2)^{\frac{3}{2}} + C$$

$$= \frac{4}{3} (x^3+2) \sqrt{x^3+2} + C$$

Exer: $\int \frac{x}{\sqrt{x^2-5}} dx = \int x(x^2-5)^{-\frac{1}{2}} dx$, com $x > \sqrt{5}$

$g(x) = x^{-\frac{1}{2}}$
 $g'(x) = 2x^{-\frac{3}{2}}$

$f(x) = x^2 - 5$
 $f'(x) = 2x$

$\rightarrow \frac{1}{2} \int 2 \cdot x (x^2-5)^{-\frac{1}{2}} dx = \frac{1}{2} \cdot 2 (x^2-5)^{\frac{1}{2}} + C$

$\frac{1}{2} \cdot 2 = 1$

valor

$= \sqrt{x^2-5} + C$