

Potensifunktion integraali

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

- yleistyys myös murtopotensseille ja juurille

Määrittelyjoukot:

- kun $n = 1, 2, 3, \dots$ niin $x \in \mathbb{R}$
- kun $n = \dots, -3, -2, -1$, niin $x \neq 0$
- kun n on murtoluku niin $x > 0$

$$\begin{aligned}
 \text{Exim. } \int \sqrt[3]{x} \, dx & \text{, kun } x > 0 \\
 = \int x^{\frac{1}{3}} \, dx & = \frac{1}{\frac{1}{3}+1} x^{\frac{1}{3}+1} + C \\
 & = \frac{1}{\frac{4}{3}} x^{\frac{4}{3}} + C \\
 & = \frac{3}{4} x^{\frac{4}{3}} + C \\
 & = \frac{3}{4} x \sqrt[3]{x} + C
 \end{aligned}$$

$$\left. \begin{aligned}
 \frac{1}{3} + \frac{3}{3} &= \frac{4}{3} \\
 \frac{1}{4} &= \frac{3}{4} \\
 x^{\frac{4}{3}} &= x^1 \cdot x^{\frac{1}{3}} = x \sqrt[3]{x}
 \end{aligned} \right\}$$

$$\text{Exim. } \int -\frac{3}{x\sqrt{x}} dx, x > 0$$

$$\begin{aligned} 2 \int -\frac{3}{x \cdot x^{\frac{1}{2}}} dx &= \int -\frac{3}{x^{\frac{3}{2}}} dx = \int -3x^{-\frac{3}{2}} dx \\ &= (-3) \cdot (-2) X^{-\frac{1}{2}} + C \\ &= 6X^{-\frac{1}{2}} + C \\ &= \frac{6}{\sqrt{x}} + C \\ &= \underline{\underline{\frac{6}{\sqrt{x}} + C}} \end{aligned}$$

$$\begin{aligned} -\frac{3}{2} + 1 &= -\frac{3}{2} + \frac{2}{2} = -\frac{1}{2} \\ \frac{1}{-\frac{1}{2}} &= -2 \\ X^{-\frac{1}{2}} &= \frac{1}{X^{\frac{1}{2}}} = \frac{1}{\sqrt{x}} \end{aligned}$$

6.7

Integroi funktio, kun $x > 0$.~~CAS~~

a) $\frac{1-x}{\sqrt{x}}$

b) $\frac{x-\sqrt{x}}{x} = \frac{x}{x} - \frac{\sqrt{x}}{x} = 1 - \frac{x^{\frac{1}{2}}}{x} = 1 - x^{-\frac{1}{2}}$

$$\int (1 - x^{-\frac{1}{2}}) dx = x - 2 \cdot x^{\frac{1}{2}} + C$$

$$= \underline{\underline{x - 2\sqrt{x} + C}}$$

HUOM

 $x \neq 0$

$$\int \frac{3}{x^3} dx$$

z

$$\frac{-3}{2 \cdot x^2}$$

Kaikki integraalipunktiot $\int \frac{3}{x^3} dx =$

$$\left\{ \begin{array}{l} -\frac{3}{2x^2} + C, \text{ kun } x < 0 \\ -\frac{3}{2x^2} + D, \text{ kun } x > 0 \end{array} \right.$$