21.17 Sievennä lauseke logaritmin laskusääntöjä käyttäen.

a) $\ln 6 - 2\ln 3 + \ln\frac{1}{2}$

b) $\ln e^3 + \ln\frac{1}{e}$

c) $\ln\frac{x+1}{x} + \ln x$, missä $x > 0$

a) $\ln 6 - \ln 3^2 + \ln\frac{1}{2} =$

$\ln\frac{6}{9} + \ln\frac{1}{2} =$

$\ln\left(\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\right) = \ln\frac{1}{3}$

$= \ln 3^{-1}$

$= \underline{-\ln 3}$

b) $\ln e^3 + \ln e^{-1} =$

$3 + (-1) = 2$

c) $\ln\frac{x+1}{x} + \ln x =$

$\ln(x+1) - \cancel{\ln x} + \cancel{\ln x} =$

$\underline{\ln(x+1)}$

Logaritmin derivaatta

Kirjoitetaan $x = e^{\ln x}$

$$Dx = De^{\ln x}$$

$$1 = \underbrace{e^{\ln x}}_{x} \cdot D\ln x \quad \| : x$$

$$\boxed{D\ln x = \frac{1}{x}}$$

$$\boxed{D\ln(f(x)) = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}}$$

$$\log_a x = \frac{\ln x}{\ln a} = \frac{1}{\underbrace{\ln a}_{\text{vakioluku}}} \cdot \ln x$$

$$D\frac{1}{\ln a} \cdot \ln x = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\boxed{D\log_a x = \frac{1}{x \cdot \ln a}}$$

22.2 Derivoi, kun $x > 0$.

a) $\ln\frac{5}{x}$ **b)** $\ln\frac{5}{2x}$ **c)** $\ln x^3$

b) $\ln\frac{5}{2x} = \underbrace{\ln 5}_{\text{vakioluku}} - \ln 2x = \ln 5 - (\ln 2 + \ln x)$

$= \underbrace{\ln 5 - \ln 2}_{\text{vakioita}} - \ln x$

$D(\ln 5 - \ln 2x) = 0 - \frac{1}{2x} \cdot 2 = -\frac{1}{x}$

$D(\ln 5 - \ln 2 - \ln x) = 0 - 0 - \frac{1}{x} = -\frac{1}{x}$

22.6 Määritä funktion f derivaattafunktion nollakohdat.

a) $f(x) = x\ln x$ **b)** $f(x) = \frac{\ln x}{x}$

$(a \cdot b)' = a'b + ab'$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

$f'(x) = 0$, kun

$\ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1} = \frac{1}{e}$