21.15 Määritä funktion f derivaattafunktion nollakohdat.

a) $f(x)=x^2e^{-x}$

b) $f(x)=\dfrac{e^{-x}}{x^2}$, Mj $x \neq 0$

a)
$$f'(x) = 2x \cdot e^{-x} + x^2 \cdot e^{-x} \cdot (-1)$$
$$= \underbrace{e^{-x}}_{>0}(2x - x^2)$$
$f'(x) = 0$, kun $2x - x^2 = 0$
$$x(2-x) = 0$$
$$\underline{\underline{x = 0}} \lor 2 - x = 0$$
$$\underline{x = 2}$$

b)
$$f'(x) = \frac{-e^{-x} \cdot x^2 - e^{-x} \cdot 2x}{(x^2)^2}$$
$f'(x) = 0$, kun
$$-e^{-x} \cdot x^2 - e^{-x} \cdot 2x = 0$$
$$\underbrace{e^{-x}}_{>0}(-x^2 - 2x) = 0$$
$$-x^2 - 2x = 0$$
$$-x(x+2) = 0$$
$$-x = 0 \lor x + 2 = 0$$
$$(x = 0) \qquad \underline{\underline{x = -2}}$$

21.17 Sievennä lauseke logaritmin laskusääntöjä käyttäen.

a) $\ln 6 - 2\ln 3 + \ln\frac{1}{2}$

b) $\ln e^3 + \ln\frac{1}{e}$

c) $\ln\frac{x+1}{x} + \ln x$, missä $x > 0$

a) $\underbrace{\ln 6 - \ln 3^2} + \ln\frac{1}{2} =$

$\ln\frac{6}{9} + \ln\frac{1}{2} =$

$\ln\frac{\cancel{6}}{\cancel{9}_3}\cdot\frac{1}{\cancel{2}_1} = \ln\frac{1}{3} = \ln 3^{-1}$

$= -\ln 3$

b) $\ln e^3 + \overbrace{\ln\frac{1}{e}}^{\ln e^{-1}} =$

$3 + (-1) = 2$

c) $\ln\frac{x+1}{x} + \ln x =$

$\ln x+1 - \ln x + \ln x =$

$\underline{\underline{\ln x+1}}$

Logaritmin derivaatta

$$x = e^{\ln x}$$

$$Dx = De^{\ln x}$$

$$1 = \underbrace{e^{\ln x}}_{x} \cdot D\ln x \quad || : x$$

$$\boxed{D\ln x = \frac{1}{x}} \Rightarrow \boxed{D\frac{1}{f(x)} = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}}$$

$$D\log_a x = D\frac{\ln x}{\ln a} = D\underbrace{\frac{1}{\ln a}}_{\text{vakio}} \cdot \ln x$$

$$= \frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{\ln a \cdot x}$$

$$\boxed{D\log_a x = \frac{1}{x\ln a}}$$

22.2 Derivoi, kun $x > 0$.

a) $\ln\frac{5}{x}$ **b)** $\ln\frac{5}{2x}$ **c)** $\ln x^3$

b) $D\ln\frac{5}{2x} = \frac{1}{\frac{5}{2x}} \cdot \frac{5}{2} \cdot (-1) \cdot x^{-2}$

$\boxed{D\ln(f(x)) = \frac{1}{f(x)} \cdot f'(x)}$

$\frac{5}{2x} = \frac{5}{2} \cdot \frac{1}{x} = \frac{5}{2}x^{-1}$

$= \frac{2\cancel{x}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{2}} \cdot \left(-\frac{1}{x^{\cancel{2}}}\right)$

$= -\frac{1}{x}$

$D\ln\frac{5}{x} = D(\ln 5 - \ln x)$

$= 0 - \frac{1}{x}$

22.6 Määritä funktion f derivaattafunktion nollakohdat.

a) $f(x) = x\ln x$ **b)** $f(x) = \frac{\ln x}{x}$

Mg $x > 0$

a) $f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$= \ln x + 1$

$f'(x) = 0$, kun

$\ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1} = \frac{1}{e}$