

5.15

Määritä yhtälön $\sin \frac{x}{2} = \sin \frac{x}{3}$

a) kaikki ratkaisut

b) välillä $[0, 2\pi]$ olevat ratkaisut.

$$\frac{x}{2} = \frac{x}{3} + m \cdot 2\pi \quad \vee \quad \frac{x}{2} = \left(\pi - \frac{x}{3}\right) + m \cdot 2\pi$$

$$\frac{x}{2} - \frac{x}{3} = m \cdot 2\pi \quad \vee \quad \frac{x}{2} + \frac{x}{3} = \pi + m \cdot 2\pi$$

$$\frac{x}{6} = m \cdot 2\pi \quad \vee \quad \frac{5x}{6} = \pi + m \cdot 2\pi \quad \parallel \cdot \frac{5}{6}$$

$$a) \quad x = m \cdot 12\pi \quad \vee$$

$$x = \frac{6\pi}{5} + m \cdot \frac{12\pi}{5}, \quad m \in \mathbb{Z}$$

$$b) \quad m=0 \Rightarrow \underline{\underline{x=0}} \quad \vee$$

$$\underline{\underline{x = \frac{6\pi}{5}}}$$

$$\left\{ \begin{array}{l} \pi = 180^\circ \\ \frac{\pi}{5} = \frac{180^\circ}{5} = 36^\circ \\ \frac{6\pi}{5} = \underline{180^\circ + 36^\circ} \\ 12 \cdot \frac{\pi}{5} = 12 \cdot 36^\circ = 360^\circ \end{array} \right.$$

5.19

Ratkaise yhtälö.



a) $\sin 2x = \cos x$

b) $\sin x = \cos 2x$

a) $\sin 2x = 2 \sin x \cos x$

$$2 \sin x \cos x = \cos x$$

$$2 \sin x \cos x - \cos x = 0 \text{ (yhtälön tekijät)}$$

$$\cos x (2 \sin x - 1) = 0 \text{ TNS}$$

$$\cos x = 0 \quad \vee \quad 2 \sin x - 1 = 0 \quad || +1 || : 2$$

$$x = \pm \frac{\pi}{2} + m \cdot 2\pi$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} + m \cdot 2\pi \quad \vee$$

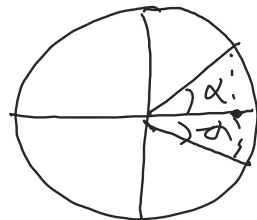
$$x = \left(\frac{\pi}{6} - \frac{\pi}{6} \right) + m \cdot 2\pi \\ = \frac{5\pi}{6} + m \cdot 2\pi$$

Perusyhtälöitä

$$\sin x = \sin y \Leftrightarrow x = y + n2\pi \vee x = \pi - y + n2\pi, n \in \mathbb{Z}$$

$$\cos x = \cos y \Leftrightarrow x = \pm y + n2\pi, n \in \mathbb{Z}$$

$$\tan x = \tan y \Leftrightarrow x = y + n\pi, n \in \mathbb{Z}$$



$$\cos x = \alpha, \text{ kun} \\ x = \pm \alpha + m \cdot 2\pi$$

Kaksinkertaiset kulmat

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$b) \sin x = \cos 2x$$

$$\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0 \quad (\text{Ratk. kaava})$$

$$a=2, \quad b=1, \quad c=-1$$

$$\sin x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{-1 \pm 3}{4} \Rightarrow$$

$$\sin x = \frac{1}{2}$$

$$\vee \sin x = -1$$

⋮
⋮
⋮

6.3

Ratkaise yhtälö.

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a) $\cos x = \frac{1}{2}$

E2

b) $\cos(x - \frac{\pi}{6}) = \frac{1}{2}$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

a) $x = \pm \frac{\pi}{3} + m \cdot 2\pi$

b) $x - \frac{\pi}{6} = \pm \frac{\pi}{3} + m \cdot 2\pi \quad \parallel + \frac{\pi}{6}$

$$x = \left(\frac{\pi}{3} + \frac{\pi}{6} \right) + m \cdot 2\pi \quad \vee \quad x = \left(-\frac{\pi}{3} + \frac{\pi}{6} \right) + m \cdot 2\pi$$

$$x = \frac{\pi}{2} + m \cdot 2\pi$$

$$x = -\frac{\pi}{6} + m \cdot 2\pi, \quad m \in \mathbb{Z}$$

6.12 Ratkaise yhtälö.



a) $\sin x \cos x + \cos x = 0$

b) $\sqrt{2} \sin x \cos x - \sin x = 0$

b) $\sin x (\sqrt{2} \cos x - 1) = 0$

yhteinen tekijä \nearrow

tulon nollasääntö

$$\sin x = 0$$

⋮

$$\sqrt{2} \cos x - 1 = 0 \quad || +1 \quad || : \sqrt{2}$$

$$\cos x = \frac{1}{\sqrt{2}}$$

⋮