

3.14

Käytä hyväksi tietoa, että $\cos \frac{\pi}{3} = \frac{1}{2}$, ja määritä ilman laskinta.

a) $\cos\left(-\frac{\pi}{3}\right) \cdot \cos \frac{2\pi}{3}$

b) $\cos \frac{2\pi}{3} : \cos \frac{4\pi}{3} = \frac{\cos 2\pi}{-\cos \frac{\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2$

c) $3 \cos\left(-\frac{2\pi}{3}\right)$

3.15

Osoita, että yhtälö on tosi.

a) $\frac{\sin 125^\circ}{\cos 35^\circ} = 1$

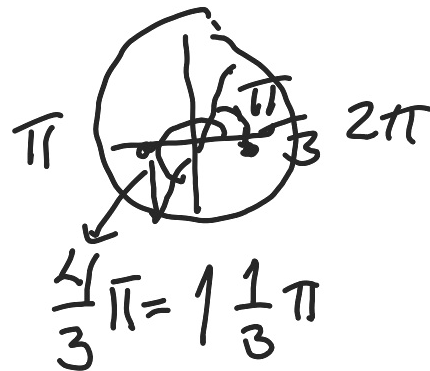
b) $\frac{\sin \frac{7\pi}{3}}{\cos \frac{\pi}{6}} = 1$

c) $\frac{\cos 5\pi - \sin \frac{17\pi}{2}}{\cos 19\pi} = 2$

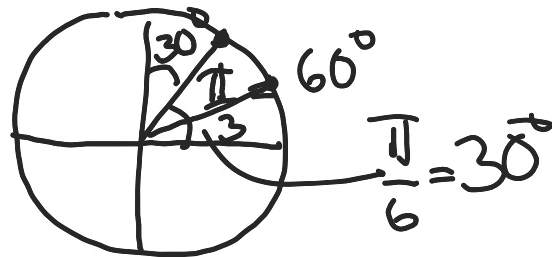
$$\frac{\sin \frac{7\pi}{3}}{\cos \frac{\pi}{6}} = \frac{\cos \frac{\pi}{6}}{\cos \frac{\pi}{6}}$$

$$\cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - x\right) = -\cos(\pi - x) = \cos(x + n2\pi)$$

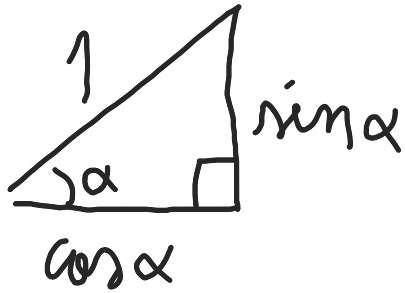
$$\cos \frac{\pi}{6} = \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \sin \frac{\pi}{3}$$



$$\frac{7\pi}{3} = 2\frac{1}{3}\pi$$



Trigonometrian perusteita



Pythagoras:

$$(\sin \alpha)^2 + (\cos \alpha)^2 = 1^2$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

HOOM!

$$(\sin \alpha)^2 = \sin^2 \alpha$$

$$(\cos \alpha)^2 = \cos^2 \alpha$$

Esim. Kulma α sijaitsee II neljänneksessä ja $\cos \alpha = \frac{1}{4}$.

Määritä $\sin \alpha$.

$$\sin^2 \alpha = 1 - \cos^2 \alpha \quad || \sqrt{\quad} \Leftrightarrow \sin \alpha \stackrel{+}{=} \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(-\frac{1}{4}\right)^2} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$$

4.4

CAS

Määritä $\sin 2\alpha$, kun $\sin \alpha = -\frac{2}{11}$ ja

$$\pi \leq \alpha \leq \frac{3\pi}{2}.$$

III neljännes!

Kaksinkertaiset kulmat

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$= 2 \cdot \left(-\frac{2}{11}\right) \cdot \left(-\frac{\sqrt{117}}{11}\right) = \frac{4\sqrt{117}}{121}$$

cos α saadaan perusteoreemalla

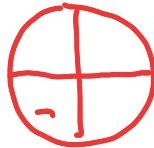
$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha \quad || \sqrt{}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \left(-\frac{2}{11}\right)^2}$$

$$= -\sqrt{\frac{117}{121}} = -\frac{\sqrt{117}}{11}$$

sin α



cos α

AOL