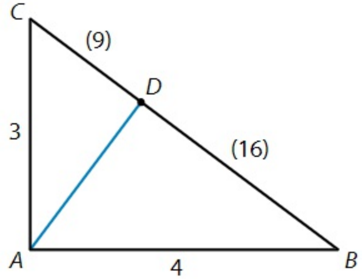
21.18 Suorakulmaisen kolmion ABC kateettien pituudet ovat 3 ja 4. Piste D jakaa hypotenuusan BC suhteessa 16 : 9. Osoita, että vektori AD on kohtisuorassa vektoria BC vastaan.



Valitaan $\overline{AB} = 4\overline{i}$

$\overline{AC} = 3\overline{j}$

Muodostetaan vektorit:

$\overline{BC} = \overline{BA} + \overline{AC} = -4\overline{i} + 3\overline{j}$

$$\begin{aligned}\overline{AD} &= \overline{AB} + \overline{BD} \\ &= \overline{AB} + \frac{16}{25}\overline{BC} \\ &= 4\overline{i} + \frac{16}{25}(-4\overline{i} + 3\overline{j}) \\ &= \frac{36}{25}\overline{i} + \frac{48}{25}\overline{j}\end{aligned}$$

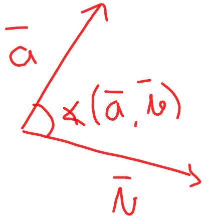
Lasketaan pistetulo

$\overline{AD} \cdot \overline{BC} = \frac{36}{25} \cdot (-4) + \frac{48}{25} \cdot 3 = 0$

Koska pistetulo $\overline{AD} \cdot \overline{BC} = 0$, niin janat

$AD \perp BC$ $\square$

Vektoreiden välinen kulma



$$\boxed{\cos(\sphericalangle(\bar{a},\bar{b})) = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}}$$

välinen kulma

$$\cos(\bar{a},\bar{b}) = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\,|\bar{b}|}, \quad 0^\circ \leq \sphericalangle(\bar{a},\bar{b}) \leq 180^\circ$$

Esim. Määritä vektoreiden $\bar{a} = 3\bar{i} + 4\bar{j}$ ja $\bar{b} = -2\bar{i} + 5\bar{j}$ välinen kulma.

Lasketaan: $\bar{a}\cdot\bar{b} = 3\cdot(-2) + 4\cdot 5 = 14$

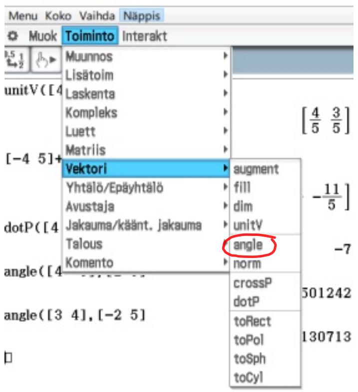
$|\bar{a}| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$|\bar{b}| = \sqrt{(-2)^2+5^2} = \sqrt{29}$

$\Rightarrow \cos(\sphericalangle(\bar{a},\bar{b})) = \frac{14}{5\cdot\sqrt{29}}$

$\sphericalangle(\bar{a},\bar{b}) = \underset{\cos^{-1}}{\arccos}\left(\frac{14}{5\sqrt{29}}\right)$

$= 58{,}67^\circ \approx \underline{\underline{58{,}7^\circ}}$



angle([3 4],[−2 5]

58.67130713

arccos((14/(5*sqrt(29))))
= 58,6713071321958330671