

11.13 Mitä pistejoukkoa yhtälö esittää?

~~CAS~~ a) $x^2 + y^2 - 4x + 6y + 9 = 0$

b) $x^2 + y^2 - 22x + 16y + 185 = 0$

c) $4x^2 + 4y^2 + 4x - 7 = 0$

$$\begin{aligned} \text{c) } 4x^2 + 4x + 4y^2 &= 7 \quad || :4 \\ x^2 + \underbrace{2 \cdot x \cdot \frac{1}{2}}_{+x} + y^2 &= \frac{7}{4} + \left(\frac{1}{2}\right)^2 \\ x^2 + x + \left(\frac{1}{2}\right)^2 + (y-0)^2 &= \frac{7}{4} + \frac{1}{4} \\ \underbrace{\left(x + \frac{1}{2}\right)^2}_{+} + (y-0)^2 &= 2 \\ \text{kp. } \left(-\frac{1}{2}, 0\right), r &= \sqrt{2} \end{aligned}$$

ympyrän leikkauspisteitä



Esim. Määritä ympyrän $x^2 + y^2 = 9$ ja suoran $y = 2x$ leikkauspisteet.

Ratkaistaan yhtälöpari

$$\begin{cases} y=2x & \text{suora} \\ x^2+y^2=9 & \text{ympyrä} \end{cases}$$

$$x^2 + (2x)^2 = 9$$

$$x^2 + 4x^2 = 9$$

$$5x^2 = 9 \quad || :5$$

$$x^2 = \frac{9}{5} \quad || \sqrt{\quad}$$

$$x = \pm \sqrt{\frac{9}{5}} = \pm \frac{3}{\sqrt{5}}$$

$$\text{kun } x = \frac{3}{\sqrt{5}} \text{ niin } y = 2 \cdot \frac{3}{\sqrt{5}} = \frac{6}{\sqrt{5}} \Rightarrow \text{piste } \left(\frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}} \right)$$

$$\text{" } x = -\frac{3}{\sqrt{5}} \text{ " } y = 2 \cdot \left(-\frac{3}{\sqrt{5}} \right) = -\frac{6}{\sqrt{5}} \Rightarrow \text{piste } \left(-\frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}} \right)$$

$$\begin{cases} y=2x \\ x^2+y^2=9 \end{cases} \Big|_{x,y}$$
$$\left\{ \left\{ x = \frac{-3\sqrt{5}}{5}, y = \frac{-6\sqrt{5}}{5} \right\}, \left\{ x = \frac{3\sqrt{5}}{5}, y = \frac{6\sqrt{5}}{5} \right\} \right\}$$

12.4 Määritä ympyröiden

~~CAS~~ $x^2 + y^2 - 8x - 4y + 15 = 0$ ja

$x^2 + y^2 + 8x - 12y + 7 = 0$

leikkauspisteet.

$$\begin{cases} x^2 + y^2 - 8x - 4y + 15 = 0 \\ x^2 + y^2 + 8x - 12y + 7 = 0 \end{cases}$$

$$-16x + 8y + 8 = 0$$

$$8y = 16x - 8 \quad || : 8$$

$$y = 2x - 1 \quad \text{inj.}^*$$

$$x^2 + (2x-1)^2 - 8x - 4(2x-1) + 15 = 0$$

$$x^2 + 4x^2 - 4x + 1 - 8x - 8x + 4 + 15 = 0$$

$$5x^2 - 20x + 20 = 0 \quad || : 5$$

$$x^2 - 4x + 4 = 0$$

$$a=1, b=-4, c=4$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{4 \pm \sqrt{0}}{2} = 2 \quad \text{inj.}^*$$

$$y = 2 \cdot 2 - 1 = 3$$

~~$$x^2 + y^2 - 8 \cdot 2 - 4y + 15 = 0$$~~

~~$$y^2 - 4y + 3 = 0$$~~

~~$$4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2}$$~~

~~$$y = \frac{4 \pm 2}{2} = \frac{4 \pm 2}{2}$$~~

~~$$y = \frac{6}{2} = 3 \vee y = \frac{2}{2} = 1$$~~

Vast. leikkauspisteet

$(2, 3)$ ja ~~$(2, 1)$~~

Ratkaisukaava:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- eq1: $x^2 + y^2 - 8x - 4y + 15 = 0$
- eq2: $x^2 + y^2 + 8x - 12y + 7 = 0$
- + Syöttökenttä...

