21.19 Ratkaise yhtälö.

CAS

a) $\frac{1}{x-1} - \frac{2}{x+1} = \frac{x+3}{x^2-1}$

b) $\frac{x}{x-1} - \frac{x+1}{(x-1)^2} = -1$

$(x+1)(x-1)$ Mj. $x \neq \pm 1$

a) $\frac{1}{x-1} - \frac{2}{x+1} = \frac{x+3}{x^2-1}$ | $\cdot (x^2-1)$

$\frac{x+1}{x^2-1} - \frac{2x-2}{x^2-1} = \frac{x+3}{x^2-1}$

$x+1-(2x-2) = x+3$

$x+1-2x+2 = x+3$

$-2x = 0 \;|:(-2)$

$x = 0$

b) Mj. $x-1 \neq 0$

$x \neq 1$

$\frac{x}{x-1} - \frac{x+1}{(x-1)^2} = -1$ | $\cdot (x-1)^2$

$\frac{x^2-x}{(x-1)^2} - \frac{x+1}{(x-1)^2} = \frac{-(x-1)^2}{(x-1)^2}$

$x^2-x-(x+1) = -(x^2-2x+1)$

$x^2-x-x-1 = -x^2+2x-1$

$2x^2-4x = 0$

$2x(x-2) = 0$

$2x = 0 \vee x-2 = 0$

$x = 0$ $\quad x = 2$

Juuripunktio

- on muotoa $\sqrt[n]{f(x)}$
  - kun n on parillinen Mj. $f(x) \geq 0$
  - kun n on pariton Mj. $\mathbb{R}$

Esim a) $g(x) = \sqrt{6x+3}$, Mj. $6x+3 \geq 0$

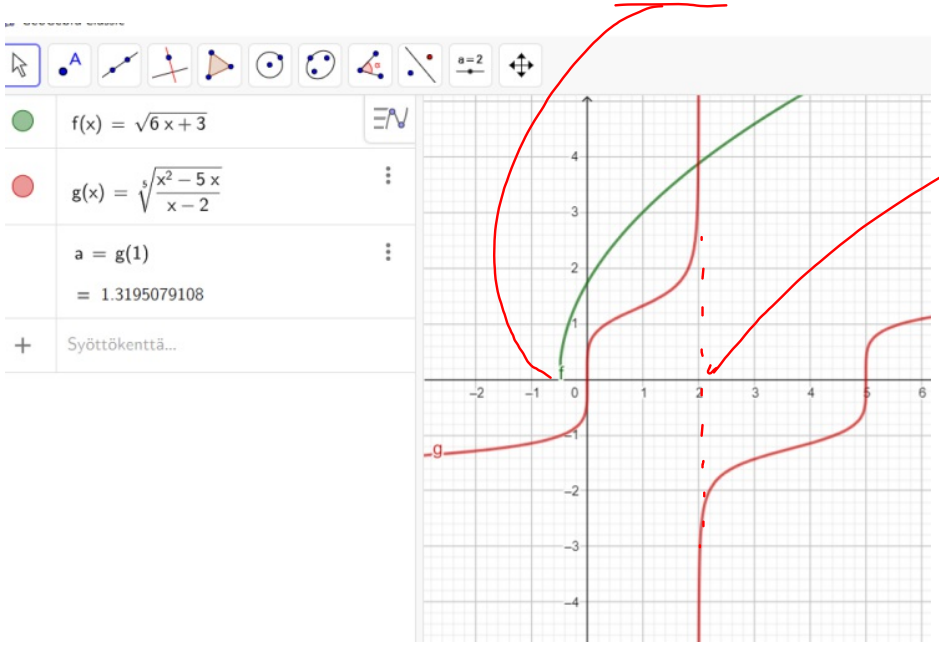
$6x \geq -3$

$x \geq -\frac{1}{2}$

b) $5\sqrt{\frac{x^2-5x}{x-2}} = h(x)$

Mj. $x-2 \neq 0$

$x \neq 2$



Neliöjuuri yhtälö

Perustelu:

$\sqrt{x} = a$, $\begin{cases} x \geq 0 \\ a \geq 0 \end{cases}$

Esim. Ratkaise yhtälö

a) $\sqrt{2x+4} = 3 \quad |()^2$

Määr. $2x+4 \geq 0$

$2x \geq -4$

$x \geq -2$

$2x + 4 = 9$

$2x = 5 \quad |:2$

$x = \frac{5}{2}$

TAI kokeillaan toteuttaako varkaus yhtälön $\sqrt{2 \cdot \frac{5}{2} + 4} = 3$ (OK)

b) $\sqrt{4x^2 - 5} = 2x + 3 \quad |()^2$

$4x^2 - 5 = (2x+3)^2$

$4x^2 - 5 = (2x)^2 + 2 \cdot 2x \cdot 3 + 3^2$

$4x^2 - 5 = 4x^2 + 12x + 9$

$-12x = 14$

$x = -\frac{14}{12} = -\frac{7}{6}$

Tarkistus $\sqrt{4 \cdot (-\frac{7}{6})^2 - 5} = \frac{2}{3}$, $2 \cdot (-\frac{7}{6}) + 3 = \frac{2}{3}$