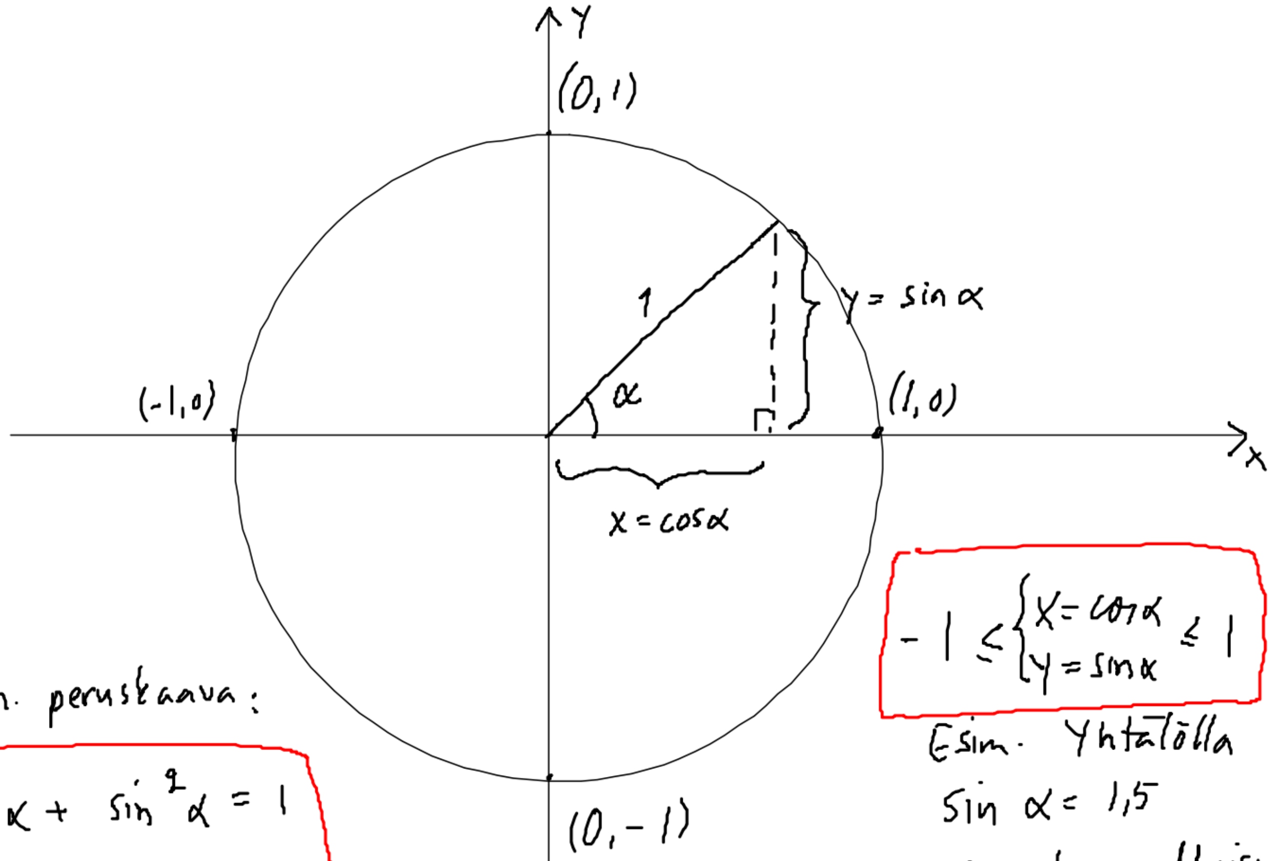
Sinin ja kosinin ominaisuuksia



$$\boxed{-1 \le \begin{cases} x=\cos\alpha \\ y=\sin\alpha \end{cases} \le 1}$$

Trigon. peruskaava:

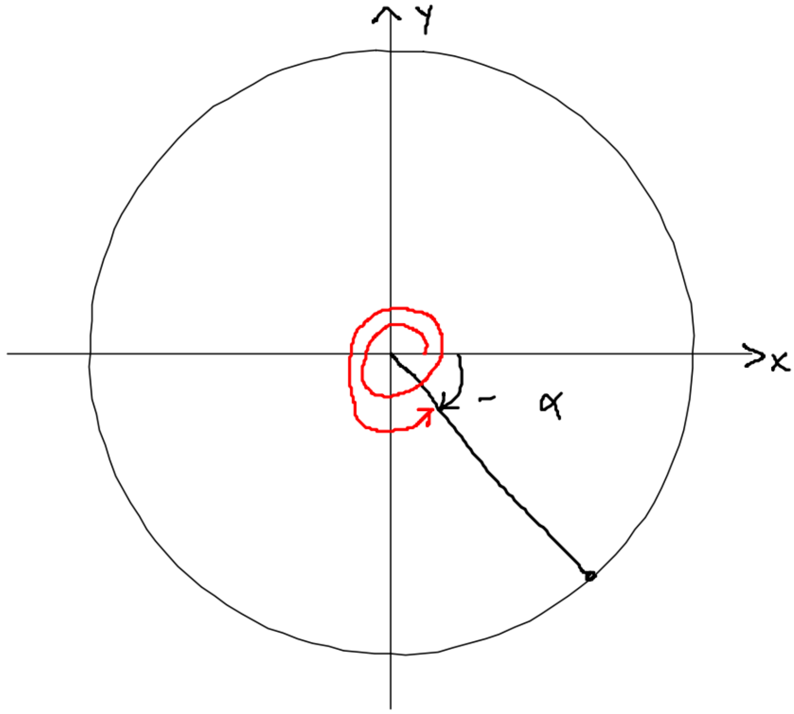
$$\boxed{\cos^2\alpha + \sin^2\alpha = 1}$$

→ $\cos^2\alpha = 1 - \sin^2\alpha$ jne.

Esim. Yhtälöllä

$\sin\alpha = 1{,}5$

ei ole ratkaisua.



Samaan kehäpisteeseen päädytään myös kulmalla $360° - \alpha$ tai $2 \cdot 360° - \alpha$ jne. tai $-360° - \alpha$ tai $-10 \cdot 360° - \alpha$ jne.

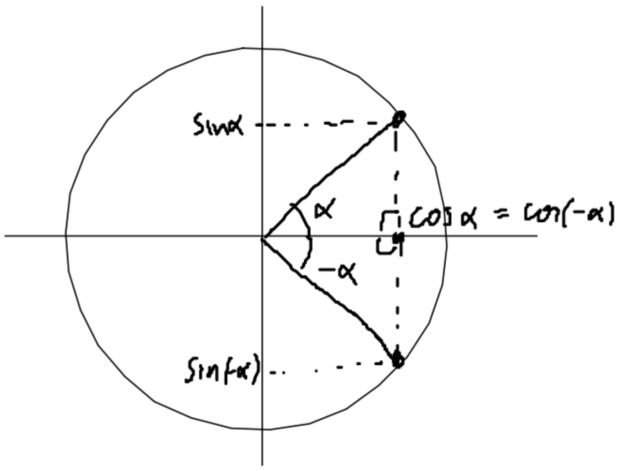
Eli

n kokon. luku

$$\boxed{\begin{aligned} \sin \alpha &= \sin(\alpha + n \cdot 360°) \\ \cos \alpha &= \cos(\alpha + n \cdot 360°) \end{aligned}}$$

$360° = 2\pi$ on sinin/kosinin <u>jakso</u>

Vastakulmat



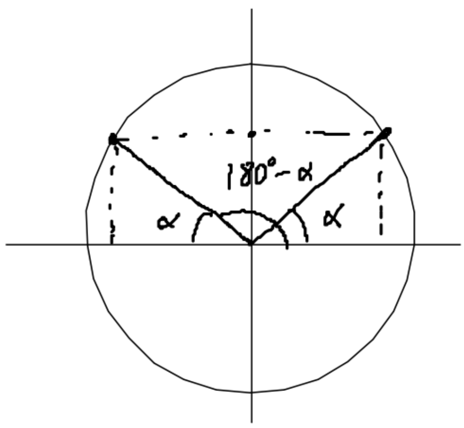
$$\boxed{\cos(-\alpha) = \cos\alpha}$$

$$\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\boxed{\sin(-\alpha) = -\sin\alpha}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

Vieruskulmat



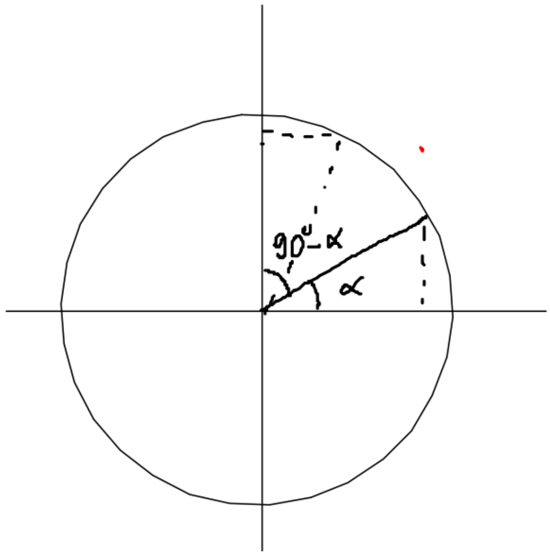
$$\boxed{\sin(180^\circ - \alpha) = \sin\alpha}$$

$$\boxed{\cos(180^\circ - \alpha) = -\cos\alpha}$$

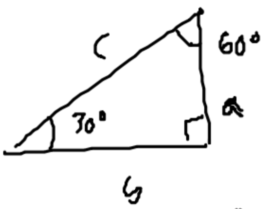
$$\cos 150^\circ = -\cos 30^\circ$$

Komplementtikulmat

(summa $90° = \frac{\pi}{2}$)



MAA3:



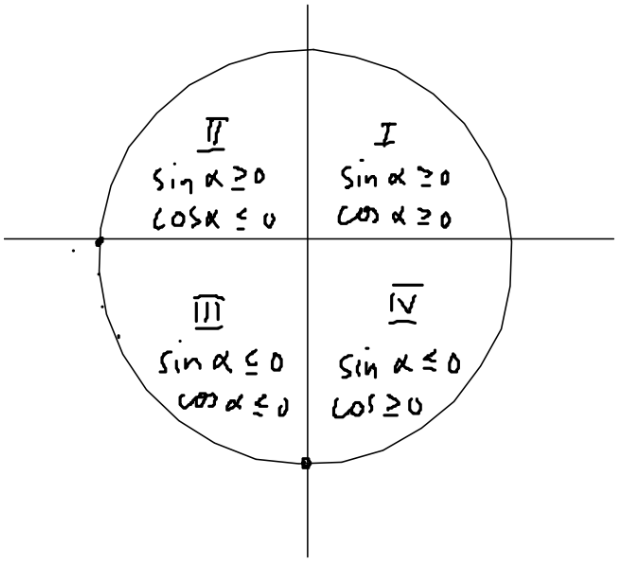
$\sin 30° = \frac{a}{c}$

$\cos 60° = \frac{a}{c}$

$$\boxed{\sin \alpha = \cos(90° - \alpha)}$$

$$\boxed{\cos \alpha = \sin(90° - \alpha)}$$

Kaksinkertaisen kulman kaavat kirjassa s.21.



238. $\sin x = -\frac{1}{5}$, $\pi < x < \frac{3\pi}{2}$ ← 3. neljännes!

$(180° < x < 270°)$

$\sin^2 x + \cos^2 x = 1$

$\left(-\frac{1}{5}\right)^2 + \cos^2 x = 1$

$\cos^2 x = 1 - \frac{1}{25}$

$\cos^2 x = \frac{24}{25} \quad \| \sqrt{}$

$\cos x = (\pm)\sqrt{\frac{24}{25}}$

V: $\cos x = -\sqrt{\frac{24}{25}}$

240. $\boxed{\text{MAOL: } \cos 2x = 2\cos^2 x - 1}$ $\cos x = \frac{1}{5}$

$$\cos 2x = 2 \cdot \left(\frac{1}{5}\right)^2 - 1 = \frac{2}{25} - 1 = \frac{2-25}{25} = -\frac{23}{25}$$

236. b) $\cos\left(-\frac{91\pi}{10}\right) = \cos\left(-\frac{11}{10}\pi - \underbrace{4 \cdot 2\pi}_{}\right)$

täydet ymp. ei merkitse mitään

$9{,}1 = \underline{4 \cdot 2} + 1{,}1$

$$\overset{\text{Jaksoll.}}{=} \cos\left(-\frac{11}{10}\pi\right)$$

$$\overset{\text{vastak.}}{=} \cos\left(\frac{11}{10}\pi\right) \overset{\text{MAOL 16}}{=} -\cos\left(\pi - \frac{11}{10}\pi\right)$$

$$= -\cos\left(-\frac{1}{10}\pi\right) \overset{\text{parill.}}{=} -\cos\frac{\pi}{10} = -\frac{\sqrt{10+2\sqrt{5}}}{4}$$

237. b) $\cos(\pi-\alpha) - \sin(\alpha-\pi) + \sin\left(\frac{\pi}{2}-\alpha\right)$

$= \cancel{-\cos\alpha} - \sin(\alpha-\pi) + \cancel{\cos\alpha}$

$= -\sin(\alpha-\pi) = \sin(\pi-\alpha) = \sin\alpha$

228. b) $\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

c) $\sin 1770° = \sin(4\cdot 360° + 330°)$

$= \sin 330° = -\frac{1}{2}$

S. 32 : 226, 227, 228, 229, 232, 233

229. $\sin 210^\circ = \sin(180^\circ - (-30^\circ))$

$= \sin(-30^\circ)$

$= -\sin(30^\circ)$

$210^\circ + \alpha = 180^\circ$

$\alpha = -30^\circ$

suplem. kulma voi olla negatiivinen!

b) $\frac{7}{6}\pi + \alpha = \pi$

$\alpha = -\frac{1}{6}\pi$

c) $-\frac{11\pi}{4} + \alpha = \pi$

$\alpha = \frac{15}{4}\pi$

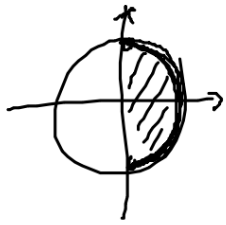
$\sin\left(-\frac{11\pi}{4}\right) = \sin\left(\frac{15}{4}\pi\right)$

$= \sin\left(2\pi + \frac{7}{4}\pi\right) = \sin\frac{7}{4}\pi = -\frac{1}{\sqrt{2}}$

232. Ratk. x: $\sin^2 x + \cos^2 x = 1$

$$\left(\frac{1}{3}\right)^2 + \cos^2 x = 1$$

$$\cos x = \pm\sqrt{1-\frac{1}{9}} = \left(\frac{+}{-}\right)\sqrt{\frac{8}{9}}$$



x posit.

Ratk. y: $\sin^2 y + \cos^2 y = 1$

$$\sin^2 y + \left(-\frac{1}{4}\right)^2 = 1$$

Y negat

$$\sin y = \pm\sqrt{1-\frac{1}{16}} = -\sqrt{\frac{15}{16}}$$

MAOL 37

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$= \frac{1}{3}\cdot\left(-\frac{1}{4}\right) + \frac{\sqrt{8}}{3}\cdot\left(-\frac{\sqrt{15}}{4}\right) = \cdots$$