Eksponenttifunktion derivaatta

$$D e^x = e^x \qquad x \in \mathbb{R}$$

$$D a^x = a^x \ln a \qquad a^x > 0$$

Esim. a) $D\ 2e^x - x^3 = 2e^x - 3x^2$

b) $D\ e^{2x} = e^{2x} \cdot 2 = 2e^{2x}$

↑ yhd. funktio: $s(x) = 2x$, $u(x) = e^x$

$$D\ u(s(x)) = u'(s(x)) \cdot s'(x)$$

c) D $x^2 \cdot e^{-x^2+3x}$

Tulon ja yhd. funktion deriv.

$= 2x \cdot e^{-x^2+3x} + x^2 \cdot e^{-x^2+3x} \cdot (-2x+3)$

$= x\, e^{-x^2+3x} \left(2 + x \cdot (-2x+3)\right)$

$= x \cdot e^{-x^2+3x} \cdot \left(-2x^2+3x+2\right) = 0$

$x = 0$ tai $-2x^2+3x+2=0$

$e^{-x^2+3x} > 0$ aina

813. $f(x) = x^2 e^{2x} + 1$, $x \in \mathbb{R}$

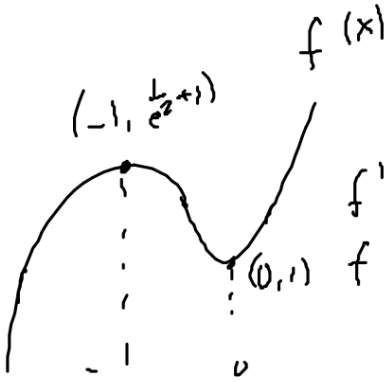
$f'(x) = 2x e^{2x} + x^2 e^{2x} \cdot 2$

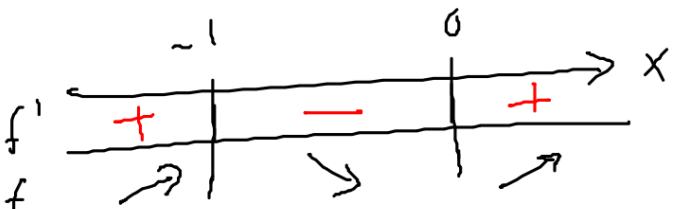
$= 2 x e^{2x} (1 + x) = 0$

$2xe^{2x} > 0$ ainas

$x = 0$ tai $1 + x = 0$

$x = -1$





$\forall: -1 \leq x \leq 0$

max. kohta
max. väärtus
$= f(-1) = \frac{1}{e^2} + 1$

min. kohta
min. arvo $f(0) = 1$

816. tangentin kulmak. pisteessä $x=a$ = $f'(a)$

$$f(x) = e^{2-x} + 4x$$

$$f'(x) = e^{2-x} \cdot (-1) + 4 = 3$$

$$-e^{2-x} = -1 \quad /\!/ : (-1)$$

$$e^{2-x} = 1 \quad /\!/ \ln$$

$$2 - x = \frac{\ln 1}{\ln e}$$

$$2 - x = 0$$

$$V: \begin{cases} x = 2 \\ y = f(2) = e^0 + 8 = 9 \end{cases}$$

S. 109: 801, 803, 805, } De^x

809, 811 } Da^x

814, 815

823. $+1{,}5\,\%$ $\xrightarrow{100\,\% + 1{,}5\,\% = 101{,}5\,\%}$ $1{,}015$

a) $4{,}03 \cdot 1{,}015^{x} = 5 \quad \| : 4{,}03$

v. 1950 $f(x) =$

$1{,}015^{x} = 1{,}24069 \quad \| \lg$

$x = \dfrac{\lg 1{,}24069}{\lg 1{,}015} \approx 14{,}5$

V: v. 1965

b) v. 2050: $4{,}03 \cdot 1{,}015^{100} \approx 18$ (milj.)

kasvunopeus v. 2050 $= f'(100)$

$f'(x) = 4{,}03 \cdot 1{,}015^{x} \cdot \ln 1{,}015$

$f'(100) = 0{,}266$ (milj./v.)

803. a) $f(x) = e^{-\sqrt{x}} = e^{-x^{\frac{1}{2}}}$ ← sisäf.

$$f'(x) = e^{-x^{\frac{1}{2}}} \cdot \left(-\tfrac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= e^{-\sqrt{x}} \cdot \left(-\frac{1}{2\sqrt{x}}\right)$$

$$= -\frac{e^{-\sqrt{x}}}{2\sqrt{x}}$$

b) $g(x) = x^2 e^x = 2x e^x + x^2 e^x = x e^x (x+2)$

$$\boxed{D\,fg = f'g + fg'}$$

c) $h(x) = (x-1)e^{-2x} = e^{-2x} + (x-1)e^{-2x} \cdot (-2)$

$$= e^{-2x}\left(1 + (x-1)(-2)\right) = \ldots \quad (= -2x+3)$$

D $\frac{f}{g} = \frac{f'g - g'f}{g^2}$

804. b) $g(x) = e^{\frac{3}{x}}$ $\quad x \neq 0$

$g'(x) = e^{\frac{3}{x}} \cdot D\left(\frac{3}{x}\right)$

$= e^{\frac{3}{x}} \cdot \left(\frac{0 \cdot x - 1 \cdot 3}{x^2}\right)$

$= -\frac{3e^{\frac{3}{x}}}{x^2}$

805. b) $f(x) = 5 - 2e^{-x} - 2e^{x+1}$

$f'(x) = 0 - 2e^{-x}(-1) - 2e^{x+1} \cdot 1$

$= 2e^{-x} - 2e^{x+1} = 0$

$\frac{2}{e^x} = \frac{2e^{x+1}}{1}$, $e^x > 0$

$a^m \cdot a^n = a^{m+n}$

$e^x \cdot e^{x+1} = 1$

$e^{2x+1} = 1 \quad \| \lg$

$2x+1 = \frac{\lg 1}{\lg e}$

$2x+1 = 0$

$x = -\frac{1}{2}$

805. a) $f(x) = e^{x-x^3}$

$f'(x) = e^{x-x^3} \cdot D(x-x^3)$

$= e^{x-x^3} \cdot (1-3x^2) = 0$

$e^{x-x^3} > 0$ ama $(1-3x^2) = 0$

$1 - 3x^2 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

809. b) $D\,2^{\pi x^2} = 2^{\pi x^2} \cdot \ln 2 \cdot D(\pi x^2)$

$= 2^{\pi x^2} \ln 2 \cdot 2\pi x$

$D\,u(s(x)) = u'(s(x)) \cdot s'(x)$