MAA7 Mallikokeen vastaukset

1. paraabelit aina muotoa $y = ax^2 + bx + c$. Etsitään a, b, c:

$f' = 2ax + b$

$$\begin{cases} f'(2) = 0 & \text{(sivuaa } y=-4 \text{:stä kohdassa 2)} \\ f(2) = -4 & \text{(— // —)} \\ f'(5) = -3 & \text{(tangentin kk kohdassa 5 on } -3) \end{cases}$$

$$\begin{cases} 4a + b = 0 \\ 4a + 2b + c = -4 \\ 10a + b = -3 \end{cases} \rightarrow \begin{cases} a = -\frac{1}{2} \\ b = 2 \\ c = -6 \end{cases}$$

2.

a) $\lim\limits_{x \to -2} \frac{(x+1)-(-2+1)}{x-(-2)} = \lim\limits_{x \to -2} \frac{x+2}{x+2} = \lim\limits_{x \to -2} 1 = 1$

b) $\lim\limits_{x \to 1} \frac{x^4 - 1^4}{x-1} = \lim\limits_{x \to 1} \frac{(x^2-1)(x^2+1)}{x-1} = \lim\limits_{x \to 1} \frac{(x+1)(x-1)(x^2+1)}{x-1} = 2 \cdot 2 = 4$

c) $\lim\limits_{x \to 3} \frac{\frac{x}{x+1} - \left(\frac{3}{3+1}\right)}{x-3} \overset{4 \mid (x+1)}{=} \lim\limits_{x \to 3} \frac{\frac{x-3}{4(x+1)}}{x-3} = \lim\limits_{x \to 3} \frac{1}{4(x+1)} = \frac{1}{16}$

(3.) $f'(x) = 2(x-1)(x+1) + (x-1)^2 = (x-1)(2x+2+x-1) = (x-1)(3x+1) = 0$

$\rightarrow x = 1$ tai $x = -\frac{1}{3}$

	$-\frac{1}{3}$		1	
f'	+	0	−	0 +
f	↗	max.	↘	min. ↗

V: $f(-\frac{1}{3})$ = maksimi arvo = ...

$f(1)$ = minimi arvo = ...

, vähenevä kun $-\frac{1}{3} < x < 1$

(4.) $f(k) = (k \cdot 1 - 1{,}2)^2 + (k \cdot 2 - 3{,}1)^2 + (k \cdot 4 - 5{,}5)^2 = 21k^2 - 58{,}8k + 41{,}3$

MAB s 2008: 13

ylösp. aukeava paraabeli → pienin arvo huipussa

$f'(x) = 42k - 58{,}8 = 0$

$k = 1{,}4$

---

5. $f'(x)$:lla kaksi nollakohtaa → ainakin 2. asteen lauseke → alkup. $f(x)$ 3. astetta

nollak. kaksi ääri.

	-1		1	
f'	+	0	−	0 +
f	↗	ääri	↘	ääri ↗

ääri. $x^3 - 3x + a$ ← kokeile sopivia, esim. 100 ?!

$f' = 3x^2 - 3 = 0$

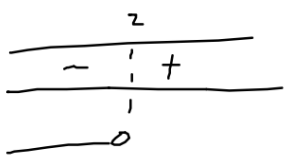
$x = \pm 1$

(6.) $f(x) = \frac{x^2+3}{x-2}$, $x \neq 2$ Kirjan tehtävä 314

a) $f(x) < 0$

$\frac{x^2+3}{x-2} < 0$

os. nollak. ei ole!



b) $f'(x) = \frac{2x(x-2) - (x^2+3)\cdot 1}{(x-2)^2} = \frac{x^2-2x-3}{(x-2)^2} = 0$

$\rightarrow x^2-2x-3=0 \rightarrow x = \frac{2 \pm \sqrt{4+12}}{2} = \begin{cases} 3 \\ -1 \end{cases}$

7.

$V = Ah = 1 \quad (dm)$

$x^2 h = 1 \rightarrow h = \frac{1}{x^2}$

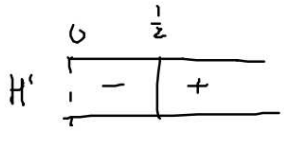
Hinta $H(x) = 16a \cdot x^2 + 4ax \cdot \frac{1}{x^2} = 16ax^2 + \frac{4a}{x}$, $x > 0$

↑ yks. hinta

$H'(x) = 32ax - \frac{4a}{x^2} = 0$

$\rightarrow 32ax^3 - 4a = 0 \rightarrow x^3 = \frac{4a}{32a} = \frac{1}{8}$

$\rightarrow x = \frac{1}{2}$



mm. arvo

kun $x = \frac{1}{2} \rightarrow h = 4 \quad (dm)$

V: pohja 5 cm, kork. 40 cm