606. $\displaystyle\int \frac{x+1}{x^3-2x^2}\,dx \quad , \qquad 0 < x < 2$

$$\frac{x+1}{x^2(x-2)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-2} \quad \Big| \; x^2(x-2)$$

$$x+1 = A(x-2) + Bx(x-2) + Cx^2$$

$$x+1 = x^2(B+C) + x(A-2B) - 2A$$

$$\begin{array}{l} x^2 \\ x \\ \end{array} \begin{cases} 0 = B+C \\ 1 = A-2B \\ 1 = -2A \end{cases} \rightarrow \begin{cases} A = -\frac{1}{2} \\ B = -\frac{3}{4} \\ C = \frac{3}{4} \end{cases}$$

$$\int \frac{x+1}{x^3-2x^2}\,dx = \int \frac{-\frac{1}{2}}{x^2}\,dx + \int \frac{-\frac{3}{4}}{x}\,dx + \int \frac{\frac{3}{4}}{x-2}\,dx = \frac{1}{2x} - \frac{3}{4}\ln x + \frac{3}{4}\ln|x-2| + C$$

$$= \frac{1}{2}x - \frac{3}{4}\ln\frac{x}{2-x} + C$$

Osittaisintegrointi

MAOL kaava 9: $D\,fg = f'g + fg' \quad \Big|\Big| \int$

$$fg + C = \int f'g\,dx + \int fg'\,dx$$

$$\rightarrow \boxed{\int f'g\,dx = fg - \int fg'\,dx}$$

614. $\int xe^x\,dx = \frac{1}{2}x^2e^x - \int \frac{1}{2}x^2e^x\,dx$

↑ ei voida laskea

Tapa 1: $f' = x \rightarrow f = \frac{1}{2}x^2$

$g = e^x \rightarrow g' = e^x$

uusi yritys:

$f' = e^x \rightarrow f = e^x$

$g = x \rightarrow g' = 1$

$= xe^x - \int 1 \cdot e^x\,dx = \underline{xe^x - e^x + C}$

619. $\int x^2 \sin x \, dx$

$g = x^2 \to g' = 2x$

$f' = \sin x \to f = -\cos x$

$= -x^2 \cos x + 2\int x \cos x \, dx$

uudelleen

$g = x \to g' = 1$

$f' = \cos x \to f = \sin x$

$= -x^2 \cos x + 2\left(x \sin x - \int 1 \cdot \sin \, dx\right)$

$= -x^2 \cos x + 2(x \sin x + \cos x) + C$

Esim. 3.

$$\int_1^e 1 \cdot \ln x \, dx \quad , \qquad x > 0$$

$$f' = 1 \to f = x$$
$$g = \ln x \to g' = \frac{1}{x}$$

$$= \Big/_1^e x \ln x - \int_1^e x \cdot \frac{1}{x} \, dx = \Big/_1^e x \ln x - \Big/_1^e x + C$$

$$= \ldots = 1$$

s. 113: 615, (MAOL: 16?)
616 →

615. $\int x e^{x^2}\,dx = \cancel{\tfrac{1}{2}x^2 e^{x^2} - \int \tfrac{1}{2}x^2 \cdot 2x e^{x^2}\,dx}$

$\cancel{f' = x \to f = \tfrac{1}{2}x^2}$

$\cancel{g = e^{x^2} \to g' = 2x e^{x^2}}$

$\cancel{f' = e^{x^2} \to ?}$

$g = x \to g' = 1$

MAOL $\boxed{\int f' e^{f}\,dx = e^{f} + C}$

$\frac{1}{2}\int \underbrace{2x}_{f'} e^{x^2}\,dx = \frac{1}{2}e^{x^2} + C$