547. $1 + a_2 + \ldots = 10$

$$S = \frac{1}{1-q} = 10 \qquad a_n = a_1 q^{n-1}$$

$$q = 0{,}9$$

$$\lg a_1 + \lg a_2 + \ldots + \lg a_{100} =$$

$$\lg\left(1 \cdot 0{,}9 \cdot 0{,}9^2 \cdot \ldots \cdot 0{,}9^{99}\right) =$$

aritm. summa $S_{99} = 99 \frac{(1+99)}{2}$

$$\lg\left(0{,}9^{1+2+\ldots+99}\right) = \lg 0{,}9^{4950}$$

$$= 4950 \cdot \lg 0{,}9$$

$$= 4950 \cdot \lg \frac{9}{10} =$$

$$= 4950 \cdot (\lg 9 - \underbrace{\lg 10}_{1})$$

Integroimismenetelmiä

- Jos $F'(x) = f(x)$ niin $F(x)$ on $f(x)$:n integraalifunktio

$$f + C \underset{\text{integ.}}{\overset{\text{deriv.}}{\rightleftarrows}} f'$$

- $\int f(x)\,dx = F(x) + C$, C = ns. integroimisvakio, se määritetään jonkun tunnetun pisteen avulla

- integ. kaavat MAOL:issa, erityisesti

(1) $$\int f'(x)\, g'\big(f(x)\big)\,dx = g\big(f(x)\big) + C$$

$$(2) \quad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$(3) \quad \int f' f^{n}\,dx = \frac{f^{n+1}}{n+1} + C$$

$$(4) \quad \int f' e^{f}\,dx = e^{f} + C$$

Esim. a) $\frac{1}{2}\cdot\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln\left(x^2+1\right) + C$

b) $\frac{1}{3}\cdot\int \underbrace{3}_{f'}\cdot\cos \underbrace{3x}_{f(x)}\,dx = \frac{1}{3}\sin 3x + C$

6.1. Murtolausekkeen integ.

- Käytä • $\int \frac{f'}{f}\,dx = \ln|f(x)| + C$

  • jaa integ. lauseke jakokulmassa

  $\longrightarrow$ integroi saatu osam. ja jakojäännös termeittäin

  • jakojäännöksen voi joutua pilkkomaan vielä osiin osamurtomenetelmällä

Esim. 1. s. 105

$$\int \frac{1}{x-x^2}\,dx \quad , \qquad x > 1$$

Osamurtomenetelmä : $\displaystyle \frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x} \quad \Big| \cdot x(1-x)$

$$1 = A(1-x) + Bx = x(-A+B) + A$$

valuot
x:t $\begin{cases} 1 = A \\ 0 = -A+B \end{cases} \quad \rightarrow \quad A=1,\ B=1$

$$\int \frac{1}{x-x^2}\,dx = \int \frac{1}{x}\,dx - \int \frac{-1}{1-x}\,dx = \ln x - \ln \underbrace{|1-x|}_{1-x<0 \text{ kun } x>1} + C$$

$$= \ln x - \ln(x-1) + C$$

Esim 2. $\displaystyle\int \frac{3x^3 - x^2 + 3x + 1}{x^3 + x}\,dx$

jakok. $\longrightarrow$ $\displaystyle\int 3 + \frac{1 - x^2}{x^3 + x}\,dx = \int 3\,dx + \int \frac{1}{x}\,dx + \int \frac{-2x}{x^2+1}\,dx$

$\downarrow$ osamurtom. $= 3x + \ln|x| - \ln(x^2+1) + C$

609. $\displaystyle\int \frac{dx}{x^2+5x+6}$, $x^2 + 5x + 6 = 0 \xrightarrow{\text{ratk. kaavalla}} x = -2$ ja -3

$x > -2$

$\displaystyle = \int \frac{1}{(x+2)(x+3)}\,dx$, osamurtomenet. $\displaystyle\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} \quad \Big| \cdot (x+2)(x+3)$

$\displaystyle = \int \frac{1}{x+2}\,dx - \int \frac{1}{x+3}\,dx$ $\rightarrow 1 = A(x+3) + B(x+2) = x(A+B) + 3A + 2B$

$\rightarrow \begin{cases} 1 = 3A + 2B \\ 0 = A + B \end{cases} \rightarrow A = 1,\ B = -1$

$= \ln|x+2| - \ln|x+3| + C$

$\displaystyle V = \ln\left(\frac{x+2}{x+3}\right) + C$

613. $\int \frac{x^4+2}{x^3-x}\,dx = \int x\,dx + \int \frac{x^2-2}{x^3-x}\,dx$

$x>1$

$= \frac{x^2-2}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$$\begin{array}{r} x \\ x^3-x \;\big)\; x^4 -2 \\ -x^4 \pm x^2 \\ \hline x^2-2 \end{array}$$

$\rightarrow A = -2$

$B = \frac{3}{2}$

$C = \frac{3}{2}$

$= \int x\,dx + \int -\frac{2}{x}\,dx + \int \frac{\frac{3}{2}}{x-1}\,dx + \int \frac{\frac{3}{2}}{x+1}\,dx$

$= \frac{1}{2}x^2 - 2\ln x + \frac{3}{2}\ln(x-1) + \frac{3}{2}\ln(x+1) + C$

$= \frac{1}{2}x^2 - 2\ln x + \frac{3}{2}\ln(x^2-1) + C = \frac{1}{2}x^2 - \ln x^2 + \ln(x^2-1)\sqrt{x^2-1} + C$

$= \frac{1}{2}x^2 + \ln\left(\frac{(x^2-1)\sqrt{x^2-1}}{x^2}\right) + C$

S. 108: 601 (b-kohta MAOL:9)
603, 605, 606

601. b) $\displaystyle\int \frac{1}{(x-2)^2}\,dx = \int \underset{\substack{\uparrow \\ f'}}{1} \cdot \underset{\substack{\uparrow \\ f^{-2}}}{(x-2)^{-2}}\,dx$

MAOL s.9

$\displaystyle = \frac{(x-2)^{-1}}{-1} + C = -\frac{1}{x-2} + C$