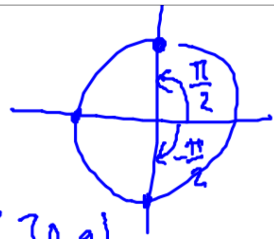
Sijoitusmenetelmä



Vaiheet:

1. Valitse korvattava, x:ää sisältävä $t = f(x) \rightarrow x = g(t)$
2. Muodosta uusi differentiaali
   $dx = g'(t)\,dt$
3. Korvaa alkup. ylä- ja alaraja uusilla
4. Integroi "uusi" integraali normaalisti

Esim. 630 a)

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

1. Sijoitus $x = \sin t$
   $\downarrow$ x:n suht. $\downarrow$ t:n suht.
2. $1\,dx = \cos t\,dt$
3. $t = \sin^{-1} x$

x	$t = \sin^{-1} x$
-1	$\sin^{-1}(-1) = -\frac{\pi}{2}$
1	$\sin^{-1}(1) = \frac{\pi}{2}$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\underbrace{1-\sin^2 t}_{\cos^2 t}} \cdot \cos t\,dt$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^2 t} \cdot \cos t \, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos t| \cdot \cos t \, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t \, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1 + \frac{1}{2} \cdot 2 \cos 2t\right) dt$$

MAOL:

$$\cos^2 t = 1 + \cos 2t$$

$$= \Big/_{-\frac{\pi}{2}}^{\frac{\pi}{2}} t + \frac{1}{2} \sin 2t = \dots = \frac{\pi}{2}$$

632. $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\,dx$

$t = 1-x^2$

$\to x^2 = 1-t$

$\to 2x\,dx = -dt$

$\to dx = -\frac{1}{2x}dt$

x	$t=1-x^2$
0	1
1	0

$= \displaystyle\int_1^0 \cancel{x^3}^{2}\sqrt{t}\cdot\left(-\frac{1}{2\cancel{x}}\right)dt$

$= \displaystyle\frac{1}{2}\int_0^1 x^2\sqrt{t}\,dt$

5.118

$= \displaystyle\frac{1}{2}\int_0^1 (1-t)\sqrt{t}\,dt = \frac{1}{2}\int_0^1 t^{\frac{1}{2}} - t^{\frac{3}{2}}\,dt = \frac{1}{2}\Big/_0^1 \frac{2}{3}t^{\frac{3}{2}} - \frac{2}{5}t^{\frac{5}{2}}$

$= \frac{2}{15}$