424. $a_n = \dfrac{2 \overset{+2}{\curvearrowright} 4 \overset{+2}{\curvearrowright} + 6 + \ldots + 2n}{n^2}$ ← os. on aritm. summa, n termiä

$a_1 = 2$, $a_n = 2n$

$\rightarrow S_n = n \cdot \dfrac{(2+2n)}{2}$

$= n \cdot \dfrac{\cancel{2}(n+1)}{\cancel{2}}$

$= n^2 + n$

$a_n = \dfrac{n^2+n}{n^2} = 1 + \dfrac{1}{n} \rightarrow 1$

kun $n \rightarrow \infty$

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415. $\lim\limits_{n\to\infty} \dfrac{2n^2+1}{3n^2-1} = \lim\limits_{n\to\infty} \dfrac{2+\frac{1}{n^2}}{3-\frac{1}{n^2}} = \dfrac{2}{3}$

$\left| \dfrac{2n^2+1}{3n^2-1}^{(3} - \dfrac{2}{3}^{(3n^2-1} \right| = \left| \dfrac{5}{9n^2-3} \right| = \dfrac{5}{9n^2-3} < 0{,}00001$

$\dfrac{5}{0{,}00001} < 9n^2 - 3$

$n^2 > 55555{,}9$

$n > 235{,}7$

V: kun $n \geq 236$

Geom. lukujono

a_1 , $a_1 q$, $a_1 q^2$, ... , $\boxed{a_n = a_1 q^{n-1}}$

$a_1 \xrightarrow{\cdot q} a_1 q \xrightarrow{\cdot q} a_1 q^2$

Jos $q > 1$: $a_n \to \infty$ kun $n \to \infty$
hajaantuu

$\boxed{q = 1}$: $a_n = a_1$ raja-arvona a_1

$\boxed{0 < q < 1}$: $a_n \to 0$ kun $n \to \infty$ eli
raja-arvona 0

$q = 0$: $a_n = 0$ raja-arvona 0

$\boxed{-1 < q < 0}$: $a_n \to 0$ kun $n \to \infty$ eli
raja-arvona 0

- $q = -1$: arvot $a_1, -a_1, \ldots$ hajaantuu
- $q < -1$: $a_n \to \pm\infty$ kun $n \to \infty$
  hajaantuu

$\longrightarrow$ Geom. jono suppenee jos

$$\boxed{-1 < q \leq 1 \quad \text{tai} \quad a_1 = 0}$$

431. $a_n = (1-x^2)^n \quad , \quad n \in \mathbb{N} = \{0, 1, 2, 3, \ldots\}$

- 0^0 ei ole määritelty $\rightarrow 1 - x^2 \neq 0$

  $x \neq \pm 1$

- on se geom. jono koska

$$q = \frac{a_{n+1}}{a_n} = \frac{(1-x^2)^{n+1}}{(1-x^2)^n} = 1 - x^2 = \text{vakio}$$

$\rightarrow$ suppenee kun $-1 < 1 - x^2 \leq 1$

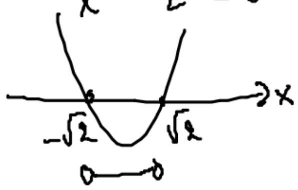
$$-1 < 1-x^2 \le 1$$

$$-1 < 1-x^2$$

$$-2 < -x^2$$

$$x^2 < 2$$

$$x^2 - 2 < 0$$



$$1-x^2 \le 1$$

$$-x^2 \le 0$$

$$x^2 \ge 0$$

totta kaikilla x:n arvoilla

V: $-\sqrt{2} < x < \sqrt{2}$, $x \neq \pm 1$

430. a) $a_n = (x^3 - 1)^n$, $n = 1, 2, 3, \ldots$ (0^0 ei ongelma)

On se geom. jono koska $q = \frac{a_{n+1}}{a_n} = \frac{(x^3-1)^{n+1}}{(x^3-1)^n}$

$= x^3 - 1 =$ vakio

$-1 < x^3 - 1 \leq 1 \quad \| +1$

$0 < x^3 \leq 2 \quad \| \sqrt[3]{\ }$ $\quad \sqrt[3]{x}$ aid. kasvava funktio

$\underline{\underline{0 < x \leq \sqrt[3]{2}}}$

Jos • $0 < x < \sqrt[3]{2}$: $\lim_{n \to \infty} a_n = \underline{\underline{0}}$

• $x = \sqrt[3]{2}$: $\lim_{n \to \infty} a_n = a_1 = \left((\sqrt[3]{2})^3 - 1\right)^1$

$= (2-1)^1 = \underline{\underline{1}}$

b) $a_n = 5(3x^2-1)^n$

Geom. jono: $q = \frac{a_{n+1}}{a_n} = \frac{\cancel{5}(3x^2-1)^{n+1}}{\cancel{5}(3x^2-1)^n} = 3x^2-1$

vakio

5.75:
428, 429
432

$$-1 < 3x^2-1 \le 1$$

$-1 < 3x^2-1$

$3x^2 > 0$

$x^2 > 0$

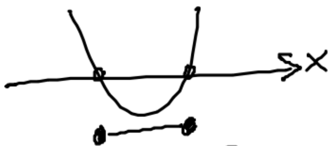
totta kun $x \ne 0$

$3x^2 - 1 \le 1$

$3x^2 - 2 \le 0$

$3x^2 - 2 = 0$

$x = \pm\sqrt{\frac{2}{3}}$



- $x = \pm\sqrt{\frac{2}{3}}$: $a_n = 5\left(3\cdot\frac{2}{3}-1\right)^n = 5\cdot 1^n \to 5$ kun $n\to\infty$
- $-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$: $a_n \to 0$ kun $n\to\infty$

V: $\underline{-\sqrt{\frac{2}{3}} \le x \le \sqrt{\frac{2}{3}}}$, $x \ne 0$

428.

$q = x - 5 = \text{vakio}$

$-1 < x - 5 \leq 1 \quad \| : +5$

$4 < x \leq 6$

$4 < x < 6 \quad : \quad \text{raja-arvo} = 0$

$x = 6 \quad : \quad \text{raja-arvo} = a_1 = 7$