212. Nimittäjällä ei ole nollak.

$$
\begin{array}{r|l}
 & x^2 - 3x - 1 \\
\hline
7x^2 - 2x + 4 & 7x^4 - 23x^3 + 3x^2 + ax + b \\
 & -7x^4 \pm 2x^3 \mp 4x^2 \\
\hline
 & -21x^3 - x^2 + ax + b \\
 & \pm 21x^3 \mp 6x^2 \pm 12x \\
\hline
 & -7x^2 + (a+12)x + b \\
 & \pm 7x^2 \mp 2x \pm 4 \\
\hline
 & \underbrace{(a+10)}_{=0}x + \underbrace{b+4}_{=0}
\end{array}
$$

$a = -10 \qquad b = -4$

$x \neq 1$

214. a) $\lim\limits_{x \to 1} \dfrac{1-x^3}{1-x} = \lim\limits_{x \to 1} \left(x^2+x+1\right) = 3$

$$
\begin{array}{r|l}
 & x^2+x+1 \\
\hline
-x+1 & -x^3 \qquad +1 \\
 & \pm x^3 \mp x^2 \\
\hline
 & -x^2+1 \\
 & \pm x^2 \mp x \\
\hline
 & -x+1 \\
 & \pm x \mp 1 \\
\hline
 & 0
\end{array}
$$

219.

$f(x) = x^3 + 1$

$f'(x) = 3x^2$

$f'(1) = 3$

$y - y_0 = k(x - x_0)$

$y - 2 = f'(1)(x - 1)$

$\begin{cases} y - 2 = 3(x-1) \Rightarrow y = 3x - 1 \\ y = x^3 + 1 \end{cases}$

$3x - 1 = x^3 + 1$

$1 - x^3 - 3x + 2 = 0$

$x - 1 = 0$

Kuranjanta $x = 1$:

$1^3 - 3 \cdot 1 + 2 = 0$

$x - 1 \overline{)\, x^3 - 3x + 2}$

$x^2 + x - 2$

$-x^3 + x^2$

$x^2 - 3x + 2$

$-x^2 + x$

$-2x + 2 = 0$

$x^2 + x - 2 = 0$

;

$x = 1$ tai -2

V: $x = 1$ tai $x = -2$

Osamurtomenetelmä

Esim. $\frac{1}{6} = \frac{1}{2} + \frac{(-1)}{3}$

↑ tekijät 2·3

- Jos os. osteluku $\geq$ nim. osteluku niin tee ensin jakolaskun jakokulmassa

222. $\frac{x^2 - x + 1}{x^3} = \frac{x^2}{x^3} - \frac{x}{x^3} + \frac{1}{x^3} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}$

227. $\dfrac{x^2+x+2}{x(x^2+x+1)} = \dfrac{A}{x} + \dfrac{B(x^2+x+1)}{x^2+x+1}$

Nim. ei voi jakaa enempää tekijöihin:

$x^2+x+1=0$

$x = \dfrac{-1 \pm \sqrt{1-4\cdot 1\cdot 1}}{2\cdot 1}$ ei nollaa.

$= \dfrac{A(x^2+x+1) + Bx}{x(x^2+x+1)} = \dfrac{Ax^2+(A+B)x+A}{x(x^2+x+1)}$

vertaa puhdista lausekkeita!

x^2:n kerroin $1 = A$ ←

x:n kerroin $1 = A+B$

vakion kerroin $2 = A$ ←

ei auttanut!

$$\frac{x^2+x+2}{x(x^2+x+1)} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$$

(x^2+x+1) $(x$

$$= \frac{Ax^2+Ax+A+Bx^2+Cx}{x(x^2+x+1)}$$

x^2:n kerroin

x:n $-\!\!\!''\!\!\!-$

vakiot

$$\begin{cases} 1 = A+B \rightarrow B=-1 \\ 1 = A+C \rightarrow C=-1 \\ 2 = A \end{cases}$$

$\therefore$ $\dfrac{x^2+x+2}{x(x^2+x+1)} = \dfrac{2}{x} + \dfrac{-x-1}{x^2+x+1}$

229. $\dfrac{3x^3+2x^2-9x+4}{x^3(x-4)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{D}{x-4}$ $\quad (\cdot x^3(x-4),\ \cdot x(x-4),\ \cdot(x-4),\ \cdot x^3)$

n.m. tekintsük

$x,\ x^2,\ x^3,\ x-4$

$$= \frac{Ax^2(x-4)+Bx(x-4)+C(x-4)+Dx^3}{x^3(x-4)}$$

$$= \frac{x^3(A+D)+x^2(-4A+B)+x(-4B+C)-4C}{x^3(x-4)}$$

$$\begin{cases} 3 = A+D & \to D=3 \\ 2 = -4A+B & \to A=0 \\ -9 = -4B+C & \to B=+2 \\ 4 = -4C & \to C=-1 \end{cases}$$

V: $\dfrac{2}{x^2}-\dfrac{1}{x^3}+\dfrac{3}{x-4}$

S. 16: 223,

224 $\left(\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}\right)$

225, 226, 228

230, 231