

## Aritm. summa

$$S_n = n \cdot \frac{a_1 + a_n}{2}$$

Tarvitset yleensä myös

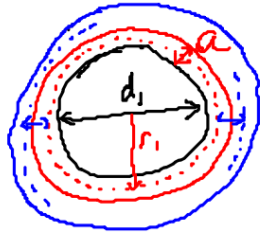
$$a_n = a_1 + (n-1)d$$

815.

$$\text{Paperia } 220 \cdot 13 = 2860 \text{ (cm)}$$

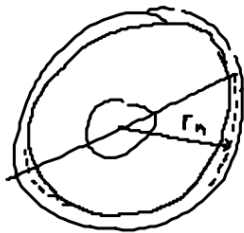
$a$  = paperin paksuus

$$d_1 = 4,5 \text{ cm}$$



ekan paperikierroksen halk. =  $4,5 + a$

$$\rightarrow \text{ekan kierroksen paperi-} \\ \text{määrä} = \pi(4,5 + a) = p_1$$



$$d_n = 12,0 \text{ cm}$$

viim. paperikierroksen  
halk. =  $12 - a$

$$\rightarrow \text{viim. kierroksen paperi-} \\ \text{määrä} = \pi(12 - a) = p_n$$

$$2860 = p_1 + p_2 + \dots + p_n$$

$$= \pi(4,5 + a) + \pi(4,5 + 3a) + \dots + \pi(12 - a)$$

aritm. summa:  $d = 2a = \text{vakio}$

$$= \pi \left( n \cdot \frac{(4,5 + a) + (12 - a)}{2} \right)$$

$$2860 = \pi \left( n \cdot \frac{16,5}{2} \right)$$

$$n \approx 110$$

V:  $n = 110$  kierrosta

## Geom. summa

- Jono  $a_n$  geom. eli  $a_n = a_1 q^{n-1}$ ,

$$q = \frac{a_{n+1}}{a_n}$$

- $S_n = a_1 + a_2 + \dots + a_n = \frac{a_1 (1 - q^n)}{1 - q} \quad q \neq 1$

Perustelu:

$$\begin{array}{l} \left\{ \begin{array}{l} S_n = a_1 + a_1/q + a_1/q^2 + \dots + a_1/q^{n-1} \quad \| \cdot (-q) \\ -qS_n = -a_1/q - a_1/q^2 - \dots - a_1/q^{n-1} - a_1 q^n \end{array} \right. \\ \hline S_n(1-q) = a_1 - a_1 q^n = a_1(1-q^n) \quad \| : (1-q) \neq 0 \\ S_n = \frac{a_1(1-q^n)}{1-q} \end{array}$$

• Jos  $q=1$  :

$$S_n = \underbrace{a_1 + a_1 + \dots + a_1}_{n \text{ kpl}} = n a_1$$

$$822. \quad a) \quad \sum_{k=0}^{50} 2^k = 2^0 + 2^1 + 2^2 + \dots + 2^{50} \\ = 1 + 2^1 + 4 + 8 + \dots + 2^{50}$$

$$\left( \begin{array}{l} a_1 = 1 \\ n = 51 \\ q = 2 \\ S_{51} = \frac{1 \cdot (1 - 2^{51})}{1 - 2} = 2^{51} - 1 \end{array} \right)$$

geom. summa,  $n=50$   
 $a_1 = 2$   
 $q = 2$

$$S_{50} = \frac{2 \cdot (1 - 2^{50})}{1 - 2}$$

$$= 1 + \left( \frac{2 - 2^{51}}{-1} \right) = 1 - 2 + 2^{51} = \underline{2^{51} - 1}$$

$$b) \sum_{n=5}^{100} (4-3n) = 4-3 \cdot 5 + 4-3 \cdot 6 + \dots + 4-3 \cdot 100$$

$$= -11 - 14 - 17 - \dots - 296$$

$$d = a_{n+1} - a_n = \dots = -3$$

arithm.  
Summa!

$$d = -3$$

$$S_n = n \frac{a_1 + a_n}{2} \rightarrow S_{96} = 96 \cdot \frac{(-11 - 296)}{2}$$

$$= 14736$$

825.

Vaihtoehto 1:

v.	V (m <sup>3</sup> )
88	100
89	$1,17 \cdot 100 - 16 = 101$
90	$1,17 \cdot 101 - 16 = 102,17$
91	$1,17 \cdot 102,17 - 16 = \dots$

V: v. 1998

1998 | 122,4

Vaihtoehto 2: symb. lastimen <sup>hankkeen</sup> laskenta  
(menu + täyhti)

Vaihtoehto 3:

$$V. \quad 88 \quad 100$$

$$+1 \quad 1,17 \cdot 100 - 16$$

$$+2 \quad 1,17(1,17 \cdot 100 - 16) - 16 = 1,17^2 \cdot 100 - 1,17 \cdot 16 - 16$$

$$+3 \quad 1,17^3 \cdot 100 - 1,17^2 \cdot 16 - 1,17 \cdot 16 - 16$$

⋮

$$+x \quad 1,17^x \cdot 100 - 1,17^{x-1} \cdot 16 - 1,17^{x-2} \cdot 16 - \dots - 1,17 \cdot 16 - 16 = 120 \quad \left. \begin{array}{l} \text{solu} \\ \text{!} \end{array} \right\}$$

Varenn puoli:

$$-16 - 16 \cdot 1,17^1 - 16 \cdot 1,17^2 - \dots - 16 \cdot 1,17^{x-1} + 1,17^x \cdot 100 = 120$$

$\underbrace{\hspace{10em}}_{\cdot (-1,17) \quad \cdot (-1,17)} \quad \text{geom. summan, } x \text{ kpl!}$

Vasen puoli:

$$\frac{-16 \cdot (1 - 1,17^x)}{1 - 1,17} + 1,17^x \cdot 100 = 120$$

$$94,118 \cdot (1 - 1,17^x) + 1,17^x \cdot 100 = 120$$

$$5,882 \cdot 1,17^x = 120 - 94,118$$

$$\vdots$$
$$x \approx 9,4$$

5. 136: 819  $\rightarrow$  824, 826

$$a_n = a_1 \cdot q^{n-1}$$

$$S_n = \frac{a_1(1 - q^n)}{(1 - q)}$$