

Sin - ja cos-yhtälön ratkaiseminen

1. Sievennä yhtälö muotoon

$$\sin x = \sin y$$

tai

$$\cos x = \cos y$$

2. Ratk. kaavat MAOL:sta

$$\underline{x = y + n2\pi}$$

$$\underline{x = \pi - y + n2\pi}$$

$$\underline{x = y + n2\pi}$$

$$\underline{x = -y + n2\pi}$$

KUMMALLAKIN KAKSI ERI RATKAISUA
+ NIIHIN TÄYDET YMPYRÄT !

$$314. \quad 4 \sin x \cdot \cos x = \sqrt{2}$$

$$\text{MAOL: } \boxed{\sin 2x = 2 \cdot \sin x \cdot \cos x} \quad \parallel : 2$$

$$\rightarrow 4 \sin x \cdot \cos x = 2 \sin 2x$$

$$\rightarrow 2 \sin 2x = \sqrt{2} \quad \parallel : 2$$

$$\sin 2x = \frac{1}{\sqrt{2}}$$

MAOL: sta tarkka arvo
 $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\sin 2x = \sin \frac{\pi}{4}$$

$$2x = \frac{\pi}{4} + n2\pi$$

$$\underline{\underline{x = \frac{\pi}{8} + n\pi}}$$

$$\text{tai } 2x = \pi - \frac{\pi}{4} + n2\pi$$

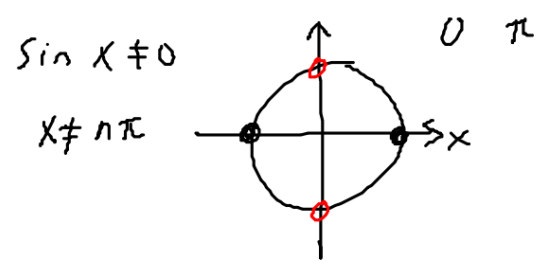
$$\text{tai } \underline{\underline{x = \frac{3\pi}{8} + n\pi}}$$

$\parallel : 2$

326. $\sin x = \frac{1}{\sin x} \quad || \cdot \sin x$

$\sin^2 x = 1 \quad || \sqrt{\quad}$

$\sin x = \pm 1$



1. $\sin x = 1$

$\sin x = \sin \frac{\pi}{2}$



$x = \frac{\pi}{2} + n2\pi$

tai $x = \pi - \frac{\pi}{2} + n2\pi$

$x = \frac{\pi}{2} + n2\pi$

2. $\sin x = -1$

$\sin x = \sin \frac{3\pi}{2}$

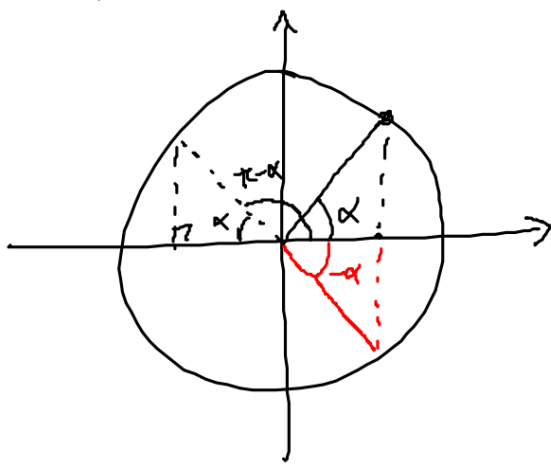


~~$x = \frac{3}{2}\pi + n2\pi$~~

tai $x = \pi - \frac{3\pi}{2} + n2\pi$

$x = -\frac{\pi}{2} + n2\pi$

Kosinin ominaisuuksia



$$x\text{-koord.} = \cos \alpha$$

$$\cos(-x) = \cos x$$

$$\cos x = \cos(x + n2\pi)$$

$$\cos x = -\cos(\pi - x)$$

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = \cos y$$



$$x = y + n2\pi$$

$$x = -y + n2\pi$$

Esim. $2 \cos x = \sqrt{3} \quad ||: 2$

$$\cos x = \frac{\sqrt{3}}{2}$$

MAOL:ista tarkka arvo

$$\cos x = \cos \frac{\pi}{6}$$

$$x = \frac{\pi}{6} + n2\pi$$

tai

$$x = -\frac{\pi}{6} + n2\pi$$

Esim. $\cos \left(x + \frac{\pi}{4} \right) = 0,7$

Laskimesta rad-lukuarvo

$$y = \cos^{-1} 0,7 = 0,79540$$

$$\cos \left(x + \frac{\pi}{4} \right) = \cos (0,79540)$$

$$x + \frac{\pi}{4} = 0,79540 + n2\pi$$

$$x = 0,010 + n2\pi$$

$$x + \frac{\pi}{4} = -0,79540 + n2\pi$$

$$x = -1,581 + n2\pi$$

$$350. \quad 2 \cos^2 x = 1 + \cos x$$

$$0 < x \leq 2\pi$$

$$2 \cos^2 x - \cos x - 1 = 0$$

Sijait. $\cos x = t$

$$\rightarrow 2t^2 - t - 1 = 0$$

$$t = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

$$\rightarrow \cos x = 1 \quad \text{tai} \quad \cos x = -\frac{1}{2}$$

$$\cos x = \cos(0)$$



$$x = 0 + n2\pi \quad \text{tai} \quad x = -0 + n2\pi$$

$$x = n2\pi$$

~~_____~~

n	$x = n2\pi$	$x = \frac{2}{3}\pi + n2\pi$	$x = -\frac{2}{3}\pi + n2\pi$
0	0 ✓	$\frac{2}{3}\pi$ ✓	$-\frac{2}{3}\pi$ ✓
1	2π ✓	$\frac{8}{3}\pi$ ✓	$\frac{4}{3}\pi$ ✓
-1	-2π ✓	$-\frac{4}{3}\pi$ ✓	✓
2	4π ✓	$\frac{14}{3}\pi$ ✓	✓
-2	-4π ✓	$-\frac{10}{3}\pi$ ✓	✓

$$\cos x = -\frac{1}{2}$$

$$\cos x = \cos\left(\frac{2\pi}{3}\right)$$

$$x = \frac{2\pi}{3} + n2\pi \quad \text{tai} \quad x = -\frac{2\pi}{3} + n2\pi$$

352.

$$\sin x = \cos\left(2x + \frac{\pi}{2}\right)$$

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\left(2x + \frac{\pi}{2}\right)$$

MAOL:

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\frac{\pi}{2} - x = 2x + \frac{\pi}{2} + n2\pi$$

$$-3x = 0 + n2\pi \quad || : -3$$

$$\underline{\underline{x = -\frac{n2\pi}{3} = \frac{n2\pi}{3}}}$$

$$\frac{\pi}{2} - x = -(2x + \frac{\pi}{2}) + n2\pi$$

$$\frac{\pi}{2} - x = -2x - \frac{\pi}{2} + n2\pi$$

$$\underline{\underline{x = -\pi + n2\pi}}$$

$$n \in \mathbb{Z}$$

S. 50: 330, 332, 334, ...