3 Measurement

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Uncertainty

There is uncertainty in every measurement. As a result, we cannot ever find the “true value” of the measured quantity. The measurement results are always approximate values with certain uncertainty.

Physical uncertainties can be expressed as absolute uncertainties or fractional uncertainties.

Using absolute uncertainty, a measurement result is expressed as

$$y = x \pm \Delta x \tag{1}$$

where x is the best estimate for the measured quantity and Δx is the absolute uncertainty in the measurement.

Fractional Uncertainty

In some cases it is not enough to know only the absolute uncertainty, but also the *fractional uncertainty* in the measurement as well.

The fractional uncertainty is

$$\Delta x_{\%} = \frac{\Delta x}{x} \tag{2}$$

where x is the best estimate for the measured quantity and Δx is the absolute uncertainty in the measurement.

Random Errors

In some cases it makes sense to repeat a measurement to have a more accurate result.

Usually, the measurement result will vary slightly in each measurement.

For example, when a tennis ball was dropped seven times from a book shelf 2.2 m above the floor, the following falling times were recorded.

t/s	0.72	0.59	0.65	0.68	0.82	0.69	0.71

The variations in the measured values of the falling time are caused by random errors in the measurement of time.

Random Errors

Definition

Random error is an error in measurement caused by factors that vary from one measurement to another in an unpredictable way.

In the tennis ball example, the time was measured manually by a stop watch.

Human reactions cause random variations to the data. The timing starts and ends differently each time: sometimes the measurer starts the stop watch a bit early, sometimes a bit late and so on. The measurer cannot see when the ball hits the ground exactly.

Random Errors (cont.)

Ideally, the measured values should be distributed normally around the real value of the measured quantity.

As a result, we can take the arithmetic average of a the measured values for the more accurate result.

Because the measured values should be centered about the real value of the measured quantity, the average gives a better estimate for the value of the quantity.

The uncertainty in the average will be less than the uncertainty in the single values.

Uncertainty in the mean

There are two ways by which we can calculate the uncertainty in the arithmetic average of the quantities.

The simplest way is to subtract the smallest value x_{min} in the data from the largest value x_{max}, and divide the result by the number of measurements N.

Definition

The uncertainty in the arithmetic average is

$$\Delta\bar{x} = \frac{x_{\mathrm{max}} - x_{\mathrm{min}}}{N} \tag{3}$$

where x_{max} is the greatest and x_{min} is the smallest value in the repeated measurement data, and N is the number of values in the data.

Uncertainty in the mean (HL)

The standard error of the mean can be used in HL work reports.

Definition

The standard error of the mean gives the uncertainty in the average of several measurements. It is defined as

$$\sigma = \frac{s}{\sqrt{N}} \quad (4)$$

where s is the *population standard deviation* and N is the number of measurements.

We calculate the standard deviation with a calculator. The detailed instructions of how to do it are in your math book and in the calculator manual.

Propagation of Error

When measurement values are used to calculate other quantities the uncertainties have to be taken into account.

In addition and subtraction, absolute uncertainties add.

In division and multiplication, fractional uncertainties add.

Accuracy

Definition

Accuracy is a measure of how close a measured value is to the real value of the measured quantity.

A measurement result is accurate, if it is close to the real value of the measured quantity.

For example, the acceleration due to gravity in Jyväskylä is $g = 9.83\,\mathrm{m\,s^{-2}}$. In an experiment, the students measured the value of $g_{\mathrm{exp}} = 9.7\,\mathrm{m\,s^{-2}}$. Because the accuracy of the measurement is $\frac{0.13}{9.83} \times 100\% \approx 1.3\%$, the result can be regarded as accurate.

Precision

Definition

Precision is an indication of the agreement among a number of measurements made in the same way.

If the spread of data is low, the measurement results are said to be precise.

For example, if in a repeated measurement the potential difference across a resistor is measured as 2.78 V, 2.75 V, 2.77 V, 2.79 V and 2.78 V, the results are precise.

In practical work, the measurement values of a quantity should be recorded with same precision (the number of decimal places should be the same in each column of data).