MAA5 3.11

1 a) $\overline{FA} = \overline{FC} + \overline{CB} + \overline{BA}$

$= -\bar{c} + (-\bar{b}) + (-\bar{a})$

$= -(\bar{a} + \bar{b} + \bar{c})$

b) $\overline{PH} = \overline{PA} + \overline{AH}$

$= \frac{1}{2}\overline{FA} + \bar{c}$

$= \frac{1}{2}\bar{c} - \frac{1}{2}\bar{a} - \frac{1}{2}\bar{b}$

c) $\overline{EI} = \overline{EH} + \overline{HA} + \overline{AI}$

$= -\bar{b} + -\bar{c} + (-\frac{1}{4}\overline{FA})$

$= \frac{1}{4}\bar{a} - \frac{3}{4}\bar{b} - \frac{3}{4}\bar{c}$

2
a) $\overline{AB} = (0-(-3))\bar{i} + (11-2)\bar{j} + (1-1)\bar{k}$

$= 3\bar{i} + 9\bar{j}$

$|\overline{AB}| = \sqrt{3^2 + 9^2}$

$= \sqrt{9(1+9)}$

$= 3\sqrt{10}$

b) $\overline{BA} = -3\bar{i} - 9\bar{j}$

$|\overline{BA}| = |AB| = 3\sqrt{10}$

$\overline{BA}^0 = \frac{1}{3\sqrt{10}}\overline{BA}$

$= -\frac{1}{\sqrt{10}}\bar{i} - \frac{3}{\sqrt{10}}\bar{j}$

$\left(= -\frac{\sqrt{10}}{10}\bar{i} - \frac{3\sqrt{10}}{10}\bar{j} \right)$

c) $\overline{AC} = (9-(-3))\overline{i} + (-2-2)\overline{j} + (5-1)\overline{k}$

$= 12\overline{i} - 4\overline{j} + 4\overline{k}$

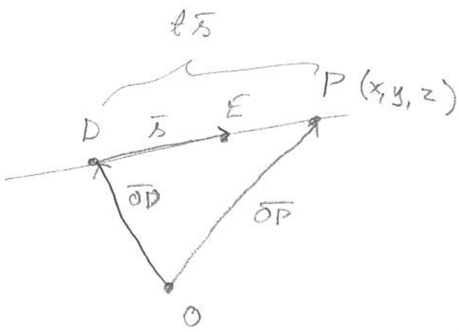
$\overline{AB} = 3\overline{i} + 9\overline{j}$

$\overline{AB} \cdot \overline{AC} = 3 \cdot 12 + 9 \cdot (-4) + 0 \cdot 4$

$= 0$

$\Rightarrow \quad \alpha = 90°$

3)



$\overline{s} = (4-1)\overline{i} + (0-2)\overline{j} + (5-3)\overline{k}$

$= 3\overline{i} - 2\overline{j} + 2\overline{k}$

$\overline{OD} = \overline{i} + 2\overline{j} + 3\overline{k}$

$\overline{OP} = \overline{OD} + t\overline{s} \qquad t \in \mathbb{R}$

$\overline{OP} = (1+3t)\overline{i} + (2-2t)\overline{j} + (3+2t)\overline{k}$

4p $\quad \begin{cases} x = 1+3t \\ y = 2-2t \\ z = 3+2t \end{cases}$

$t = \dfrac{y-2}{-2} = \dfrac{x-1}{3} = \dfrac{z-3}{2}$

4 a) $\bar{v} = \bar{i} - \bar{j}$, koska $\bar{u} \cdot \bar{v} = 1 \cdot 1 + 1 \cdot (-1) + 1 \cdot 0 = 0$

$\Rightarrow \alpha = 90^\circ$

b) $\cancel{|\bar{u}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}}$

$\cancel{\bar{u}^0 = \frac{1}{\sqrt{3}} \bar{u} = \frac{1}{\sqrt{3}} \bar{i} + \frac{1}{\sqrt{3}} \bar{j} + \frac{1}{\sqrt{3}} \bar{k}}$

$\bar{w} = -\bar{v} = -\bar{i} - \bar{j} - \bar{k}$

koska $\bar{w} = -a\bar{v}$

$(a = 1)$

c) $\bar{l} = \sqrt{3}\, \bar{i}$

koska x-akselin suuntainen, on myös xy-tasossa

5 $\overline{OP} = \overline{OA} + r\overline{AB} + s\overline{AC}$

$= (1 - r + 6s)\bar{i} + (1 - 4r + 5s)\bar{j} + (1 + r + 2s)\bar{k}$

2p? tason yhtälö laskimella: $13x - 8y - 19z + 14 = 0$

$\bar{n} = 13\bar{i} - 8\bar{j} - 19\bar{k}$

P = tason piste lähimpänä pistettä D (x, y, z)

$\overline{PD} = t\bar{n}$

$\overline{OP} = \overline{OD} + \overline{DP}$

$= 9\bar{k} - t\bar{n}$

$$\begin{cases} x = -13t \\ y = 8t \\ z = 19t + 9 \\ 13x - 8y - 19z = 14 \end{cases}$$

$x = \frac{2041}{594}$ $y = -\frac{628}{297}$ $z = \frac{2363}{594}$ $t = -\frac{157}{594}$

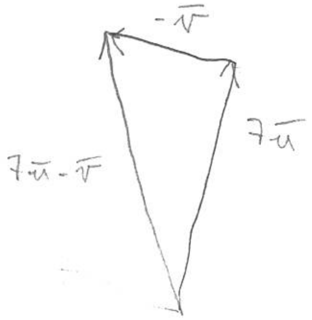
laskimella

$\Rightarrow |\overline{PD}| = \frac{157}{594} \cdot \sqrt{13^2 + 8^2 + 19^2} = \frac{157 \cdot \sqrt{66}}{198} \approx 6{,}44$

6

vektorit $7\bar{u}$, $-\bar{v}$ ja $7\bar{u}-\bar{v}$

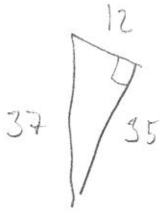
muod. kolmion



pituudet sen sivuille ovat

$|7\bar{u}| = 7 \cdot |\bar{u}| = 7 \cdot 5 = 35$

$|-\bar{v}| = |\bar{v}| = 12$



$|7\bar{u}-\bar{v}| = 37$

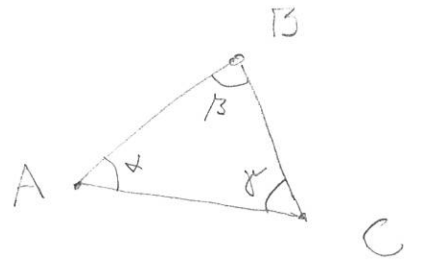
Nyt $12^2 + 35^2 = 37^2$ $\Rightarrow$ kolmiö on suorakulmainen

$\Rightarrow 7\bar{u} \perp -\bar{v}$

$\Leftrightarrow \bar{u} \perp -\bar{v}$

$\Leftrightarrow \bar{u} \perp \bar{v}$ eli $\bar{u}$ ja $\bar{v}$ ovat kohtisuorassa keskenään

(7)



$\overline{AB} = \overline{i} - 2\overline{j} + 3\overline{k}$

$\overline{AC} = 4\overline{i} + 3\overline{j} + (a-2)\overline{k}$

$\overline{BC} = 3\overline{i} + 5\overline{j} + (a-5)\overline{k}$

Mikä kulmista on suora?

$\overline{AB} \cdot \overline{AC} = 1 \cdot 4 - 2 \cdot 3 + 3 \cdot (a-2)$

$= 3a - 8$

$\alpha = 90°$, kun $a = \frac{8}{3}$

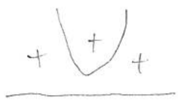
$\overline{BA} \cdot \overline{BC} = -1 \cdot 3 + 2 \cdot 5 - 3 \cdot (a-5)$

$= 22 - 3a$

$\beta = 90°$, kun $a = \frac{22}{3}$

$\overline{CB} \cdot \overline{CA} = -3 \cdot (-4) - 5 \cdot (-3) - (a-5) \cdot (-(a-2))$

$= 12 + 15 + a^2 - 7a + 10$



$a^2 - 7a + 37 = 0$

ei ratk. $\Rightarrow$ γ ei koskaan $90°$

Kun pistetulo on positiivinen, on kulma $< 90°$ (koska $\cos < 0$ kun $90°\text{–}180°$)

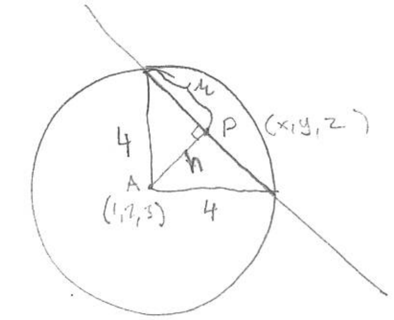
Eli $\overline{AB} \cdot \overline{AC} > 0$, kun $a > \frac{8}{3}$ $\Rightarrow$ α on terävä

$\overline{BA} \cdot \overline{BC} > 0$, kun $a < \frac{22}{3}$ $\Rightarrow$ β —"—

$\overline{CB} \cdot \overline{CA} > 0$ aina $(a \in \mathbb{R})$ $\Rightarrow$ γ —"—

Kolmio on teräväkulmainen, kun $\frac{8}{3} < a < \frac{22}{3}$

8



h = pisteen A etäisyys tasosta

P on ympyrän keskipiste

$\bar{n} = \bar{i} + 3\bar{j} - 2\bar{k}$ $|\bar{n}| = \sqrt{1^2+3^2+2^2} = \sqrt{14}$

$\overline{OP} = \overline{OA} + t\bar{n}$ $h = t|\bar{n}| = t\cdot\sqrt{14}$

$$\begin{cases} x = 1+t \\ y = 2+3t \\ z = 3-2t \\ x+3y-2z+4=0 \end{cases}$$

$\Rightarrow x = \frac{9}{14} \quad y = \frac{13}{14} \quad z = \frac{26}{7} \quad t = -\frac{5}{14}$

$\Rightarrow h = -\frac{5}{14}\sqrt{14}$

$r^2 + h^2 = 4^2$

$r = \sqrt{16 - \frac{25}{14}}$

$= \frac{\sqrt{2786}}{14}$

$\approx 3{,}77$

$\Rightarrow$ ympyrän keskipiste on $\left(\frac{9}{14}, \frac{13}{14}, \frac{26}{7}\right)$

ja säde on $\frac{\sqrt{2786}}{14} \approx 3{,}77$