8.1 Aritmeettinen summa
Arithmetic sum

-jos jono on päättyvä, voidaan sen kaikki termit laskea yhteen
If sequence in finite, its sum can be calculated

Esim. Laske lukujonon (1,3,5,7,...,99) summa
Calculate the sum.

Nyt $a_n = 2n - 1$ ja $2 \cdot 50 - 1 = 99 = a_{50}$

$$S_n = n \cdot \frac{a_1 + a_n}{2}$$

$$= 50 \cdot \frac{1 + 99}{2}$$

$$= 50 \cdot 50$$

$$= 2500$$

8.2 Geometrinen summa Geometric sum

-jos päättyvä, voidaan termit laskea yhteen
can be calculated, if finite

-jos q=1, niin jono on $(a_1, a_1, a_1, \ldots)$
ja summa on $n \cdot a_1$
-yleensä q≠1 if q=1, then sum= $n \cdot a_1$

Esim. laske jonon (1,2,4,8,16,...,1024) summa
Calculate the sum

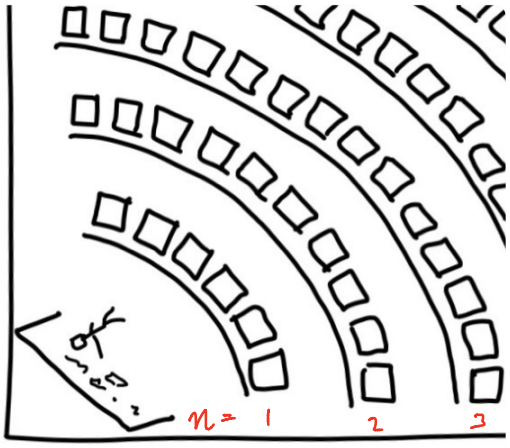
$a_1 = 1 \quad a_n = 2^{n-1} \quad 1024 = 2^{10} = a_n \quad q = 2$

$$S_n = \frac{a_1(1-q^n)}{1-q} = \frac{1 \cdot (1-2^{11})}{1-2}$$

$$= 2^{11} - 1 = 2047$$

Esim. Monellako rivillä luentosalissa on yli 300 penkkiä?

How many rows are needed, so that there are over 300 benches?



$(6, 10, 14, 18, 22, \ldots, a_n)$ $\qquad a_n = 2 + 4n$

$$S_n = n \frac{a_1 + a_n}{2} = 300$$

$$= n \cdot \frac{6 + 2 + 4n}{2} = 300$$

$$2n^2 + 4n - 300 = 0$$

$$\vdots$$

$$n = 11{,}28\ldots \quad (\text{tai } n = -13{,}28\ldots)$$

=> There has to be at least 12 rows
(11 rows gives only 286 benches, 12 gives 336)

Esim. Eräs bakteeri 7-kertaistaa lukumääränsä tasan tunnissa.
Petrimaljaan laitetaan 3 bakteeria ma klo 13.00.
Keskiviikkona klo 14 bakteerit täyttävät petrimaljan tarkalleen täyteen.
Paljonko bakteereja on silloin?
Milloin astia on tarkalleen 1/7 täynnä?
Bactery grows 7-fold in one hour.
We place 3 bacterials in a petri dish on monday 13.00. On wednesday 14.00 the petri dish is full. How many bacterials are there then?
When is the dish only 1/7 full?

Time: 2 days + 1 hour = 49 hours

Amount of bacteria is $3 \cdot 7^{49} \approx 7{,}7 \cdot 10^{41}$

$= 77077073256317663426583443167270863964821$

Dish is 1/7 full exactly at 48 hours, because the amount of bacteria grows 7-fold in 1 hour.

Esim. Antti säästää rahaa säästötilille. Joka kuukauden alussa hän laittaa tilille 150 euroa talteen. Tilin kuukaisittainen korko lähdeveron jälkeen on 0,11%.
Minkä ajan jälkeen tilillä on rahaa yli 100 000€?
Antti puts 150 euros to a savings account at the start of every month. The account has an interest rate of 0,11% /month after taxes.
When is there over 100 000€ on the account?

Money in the account at the start of

1st month: 150

2nd month: $150 + 150 \cdot 1{,}0011$

3rd month: $150 + 150 \cdot 1{,}0011 + 150 \cdot 1{,}0011^2$

4th month: $150 + 150 \cdot 1{,}0011 + 150 \cdot 1{,}0011^2 + 150 \cdot 1{,}0011^3$

...

nth month: $150 + 150 \cdot 1{,}0011 + \ldots + 150 \cdot 1{,}0011^{n-1}$

a_1 a_2 a_n

This can be considered as an geometric sum:

$q = 1{,}0011 \qquad a_1 = 150$

$$S_n = \frac{150 \cdot (1 - 1{,}0011^n)}{1 - 1{,}0011} = 100\,000$$

solve: $n \approx 500{,}37\ldots$

So in 501 months = 41 years and 9 months

Esim. Lukujonon 4. jäsen on 4 ja 7. jäsen on 8. Laske lukujonon ensimminen termi, kun lukujono on
a) aritmeettinen.
b) geometrinen.

A sequence has terms $a_4 = 4 \qquad a_7 = 8$

Calculate the first term, when the sequence is
a) arithmetic b) geometric

a) $a_4 = 4 \qquad a_7 = 8$

$a_7 = a_4 + (7-4)d$

$8 = 4 + 3d$

$d = \frac{4}{3}$

$a_4 = a_1 + (4-1)d$

$4 = a_1 + 3 \cdot \frac{4}{3}$

$a_1 = 0 \qquad \left(0, \frac{4}{3}, \frac{8}{3}, 4, \dots\right)$

b) $a_4 = 4 \qquad a_7 = 8$

$a_7 = a_4 \cdot \overbrace{q \cdot q \cdot q}^{(7-4)\text{ kpl}}$

$a_7 = a_4 \cdot q^{7-4}$

$8 = 4 \cdot q^3$

$2 = q^3$

$q = \sqrt[3]{2}$

$a_4 = a_1 \cdot q^{4-1}$

$4 = a_1 \cdot q^3$

$4 = a_1 \cdot 2$

$a_1 = 2$ $\qquad (2, 2\cdot\sqrt[3]{2}, 2\cdot(\sqrt[3]{2})^2, 4, \ldots)$

Esim. Laske lukujonon summa:

(32, -16, 8, -4, ... 1/8)

$a_1 = 32$ $\qquad q = -\frac{1}{2}$ $\qquad a_n = 32 \cdot (-\frac{1}{2})^{n-1}$ $\qquad \frac{1}{8} = a_9$

$n = 9$

$$S_n = \frac{a_1(1-q^n)}{1-q}$$

$$= \frac{32 \cdot (1-(-\frac{1}{2})^9)}{1-(-\frac{1}{2})}$$

$$= \frac{171}{8} = 21{,}375$$