Revision test 20.9.2023 [51 marks]

1. [Maximum mark: 5] 22N.2.SL.TZ0.4

geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\ 500$. [5]

Markscheme

$86.4 = 50r^3$ ***(A1)***

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere ***(A1)***

$\frac{50\left(1.2^n - 1\right)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ ***(A1)***

attempt to solve their geometric S_n inequality or equation ***(M1)***

sketch OR $n > 26.9045,\ n = 26.9$ OR $S_{26} = 28368.8$ OR
$S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ ***A1***

[5 marks]

2. [Maximum mark: 6] 22N.2.SL.TZ0.6

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a. [6]

Markscheme

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^9C_r + (ax)^{9-r} + (1)^r$.

valid approach for expansion (must be the product of a binomial coefficient with $n = 9$ and a power of ax) ***(M1)***

${}^9C_r(ax)^{9-r}(1)^r$ OR ${}^9C_{9-r}(ax)^r(1)^{9-r}$ OR
${}^9C_0(ax)^0(1)^9 + {}^9C_1(ax)^1(1)^8 + \ldots$

recognizing that the term in x^6 is needed ***(M1)***

$\frac{\text{Term in } x^6}{21x^2} = kx^4$ OR $r = 6$ OR $r = 3$ OR $9 - r = 6$

correct term or coefficient in binomial expansion (seen anywhere) ***(A1)***

${}^9C_6(ax)^6(1)^3$ OR ${}^9C_3a^6x^6$ OR $84(a^6x^6)(1)$ OR $84a^6$

EITHER

correct term in x^4 or coefficient (may be seen in equation) ***(A1)***

$\frac{{}^9C_6}{21}a^6x^4$ OR $4a^6x^4$ OR $4a^6$

Set their term in x^4 or coefficient of x^4 equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of x) ***(M1)***

$\frac{{}^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4$ OR $4a^6 = \frac{8}{7}a^5$

OR

correct term in x^6 or coefficient of x^6 (may be seen in equation) ***(A1)***

$84a^6x^6$ OR $84a^6$

set their term in x^6 or coefficient of x^6 equal to $24a^5x^6$ or $24a^5$ (do not accept other powers of x) ***(M1)***

$84a^6x^6 = 24a^5x^6$ OR $84a = 24$

THEN

$a = \frac{2}{7} \approx 0.286\,(0.285714\ldots)$ ***A1***

Note: Award **A0** for the final mark for $a = \frac{2}{7}$ and $a = 0$.

[6 marks]

3. [Maximum mark: 6] EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1 + u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1 + (3u_1 + 8)}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

Markscheme

$d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

[4 marks]

4. [Maximum mark: 6] 22M.1.SL.TZ2.4

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1.$

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

Markscheme

$x = -1$ ***A1***

[1 mark]

(a.ii) Write down the equation of the horizontal asymptote. [1]

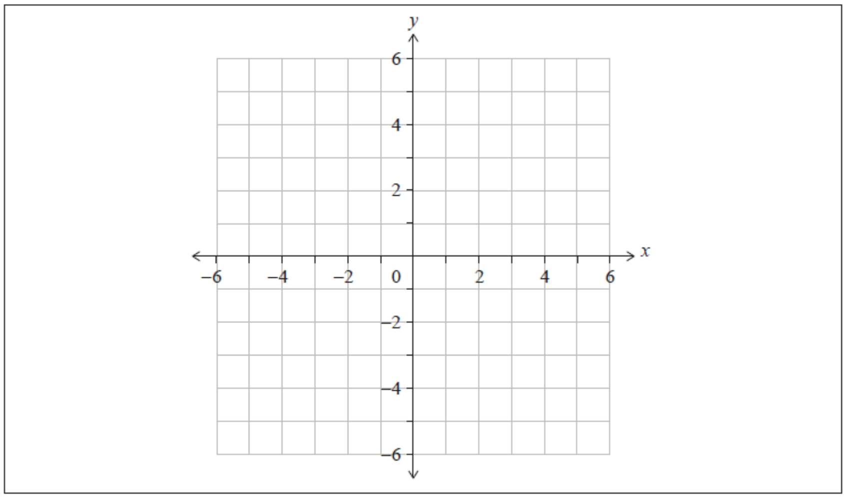
Markscheme

$y = 2$ ***A1***

[1 mark]

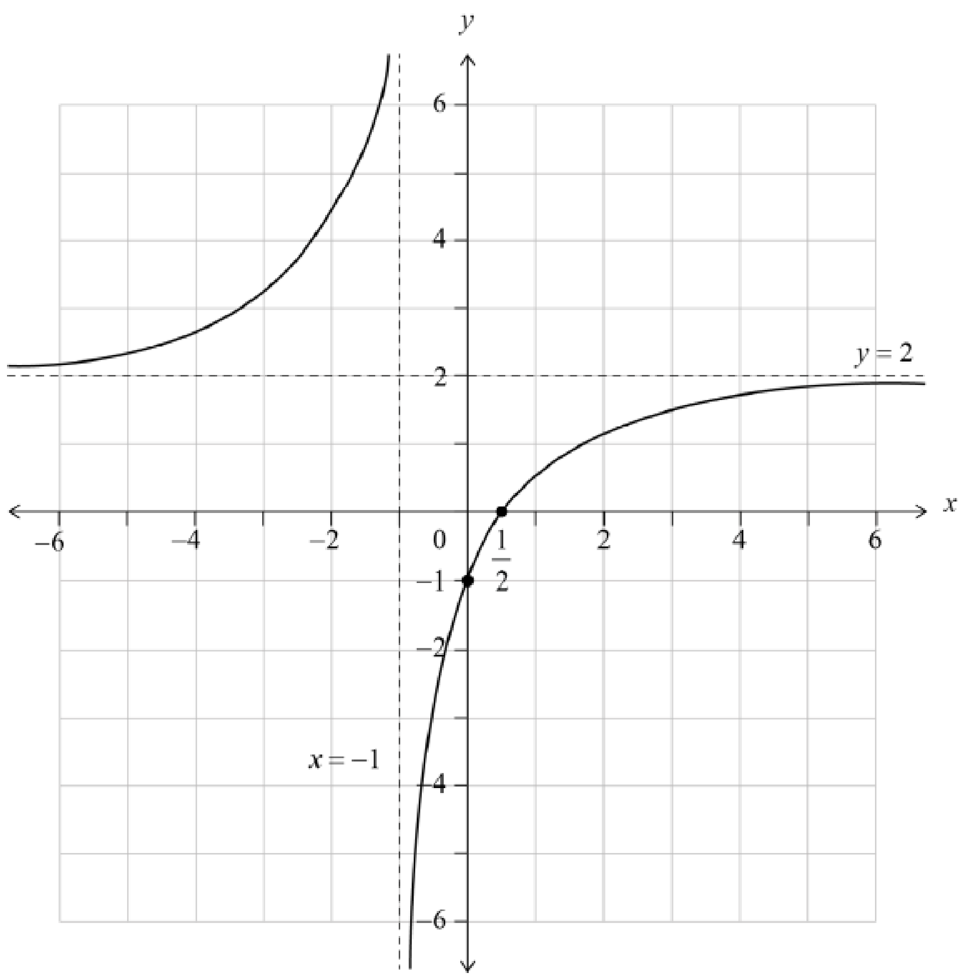
(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

Note: The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x = -1$ and $y = 2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$ ***A1A1***

[3 marks]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2.$ [1]

Markscheme

$x > \frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

5. [Maximum mark: 5] 22M.1.AHL.TZ1.6

Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all possible values of n for which the expansion has a non-zero constant term. [5]

Markscheme

EITHER

attempt to obtain the general term of the expansion

$T_{r+1} = {}_nC_r\left(8x^3\right)^{n-r}\left(-\frac{1}{2x}\right)^r$ OR $T_{r+1} = {}_nC_{n-r}\left(8x^3\right)^r\left(-\frac{1}{2x}\right)^{n-r}$ ***(M1)***

OR

recognize power of x starts at $3n$ and goes down by 4 each time ***(M1)***

THEN

recognizing the constant term when the power of x is zero (or equivalent) ***(M1)***

$r = \frac{3n}{4}$ or $n = \frac{4}{3}r$ or $3n - 4r = 0$ OR $3r - (n - r) = 0$ (or equivalent) ***A1***

r is a multiple of 3 $(r = 3, 6, 9, \ldots)$ or one correct value of n (seen anywhere) ***(A1)***

$n = 4k,\ k \in \mathbb{Z}^+$ ***A1***

Note: Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \ldots$
Do not accept $n = 4, 8, 12$

Note: Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \ldots$ and for recognizing that this pattern continues.

[5 marks]

6. [Maximum mark: 9] 21N.2.SL.TZ0.6

The sum of the first n terms of a geometric sequence is given by

$S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

Markscheme

$u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24}\left(= \frac{7}{12} = 0.583333\ldots\right)$ ***A1***

[2 marks]

(b) Find S_∞. [3]

Markscheme

$r = \frac{7}{8}(= 0.875)$ ***(A1)***

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3}(= 4.66666\ldots)$ ***A1***

[3 marks]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

Markscheme

attempt to substitute their values into the inequality or formula for S_n ***(M1)***

$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r < 0.001$ OR $S_n = \frac{\frac{7}{12}\left(1-\left(\frac{7}{8}\right)^n\right)}{\left(1-\frac{7}{8}\right)}$

attempt to solve their inequality using a table, graph or logarithms

(must be exponential) ***(M1)***

Note: Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036\ldots$ OR
$S_\infty - S_{64} = 0.000906\ldots$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683\ldots$ OR
$S_\infty - S_{64} - 0.001 = 0.0000931777\ldots$

least value is $n = 64$ ***A1***

[4 marks]

7. [Maximum mark: 5] 21M.1.SL.TZ1.3

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d. [5]

Markscheme

METHOD 1 (finding u_1 first, from S_8)

$4(u_1 + 8) = 8$ ***(A1)***

$u_1 = -6$ ***A1***

$u_1 + 7d = 8$ OR $4(2u_1 + 7d) = 8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d = 2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$ ***(A1)***

$4(u_1 + 8) = 8$ OR $4(2u_1 + 7d) = 8$ OR $u_1 = -3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1 = -6,\ d = 2$ ***A1A1***

[5 marks]

8. [Maximum mark: 5] 21M.1.SL.TZ2.4

In the expansion of $(x+k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63.

Find the possible values of k. [5]

Markscheme

EITHER

attempt to use the binomial expansion of $(x+k)^7$ ***(M1)***

${}^7C_0x^7k^0 + {}^7C_1x^6k^1 + {}^7C_2x^5k^2 + \ldots$ (or ${}^7C_0k^7x^0 + {}^7C_1k^5x^1 + {}^7C_2k^5x^2 + \ldots$)

identifying the correct term ${}^7C_2x^5k^2$ (or ${}^7C_5k^2x^5$) ***(A1)***

OR

attempt to use the general term ${}^7C_rx^rk^{7-r}$ (or ${}^7C_rk^rx^{7-r}$) ***(M1)***

$r = 2$ (or $r = 5$) ***(A1)***

THEN

${}^7C_2 = 21$ (or ${}^7C_5 = 21$ (seen anywhere) ***(A1)***

$21x^5k^2 = 63x^5$ $\left(21k^2 = 63\ ,\ k^2 = 3\right)$ ***A1***

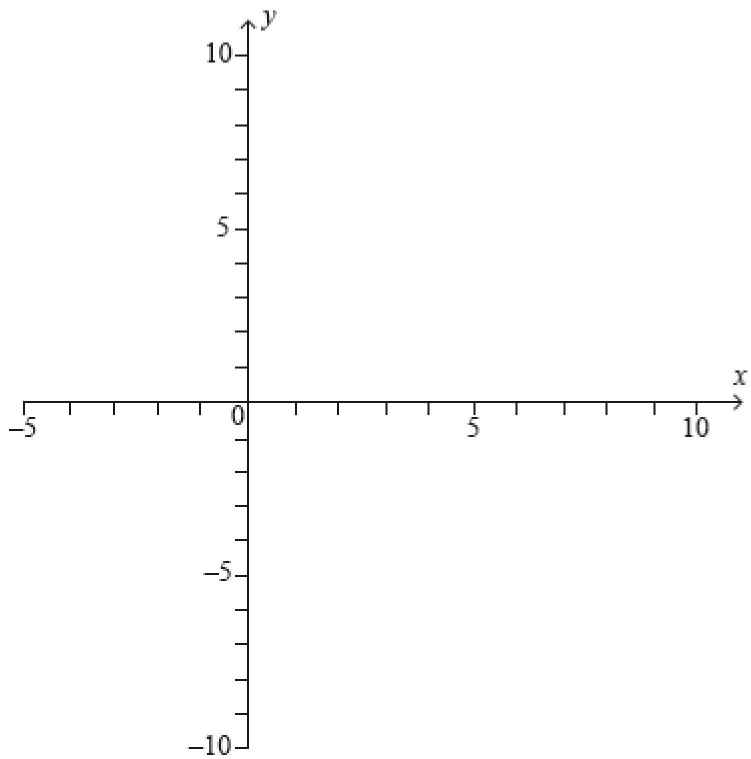
$k = \pm\sqrt{3}$ ***A1***

Note: If working shown, award ***M1A1A1A1A0*** for $k = \sqrt{3}$.

[5 marks]

9. [Maximum mark: 4] 17N.1.AHL.TZ0.H_6

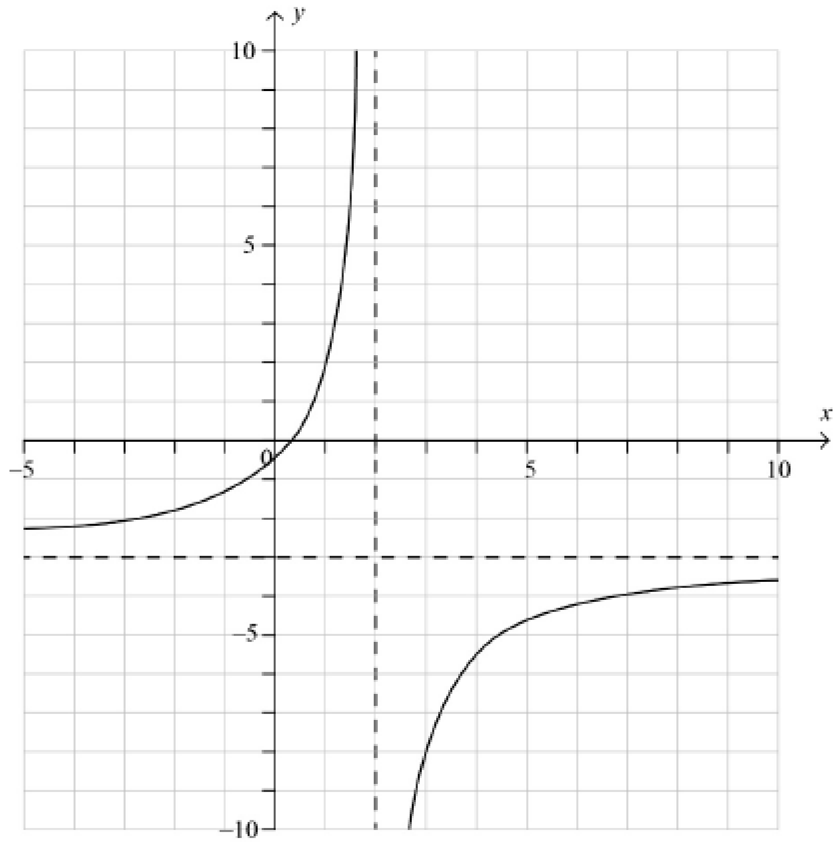
(a) Sketch the graph of $y = \frac{1-3x}{x-2}$, showing clearly any asymptotes and stating the coordinates of any points of intersection with the axes.



[4]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



correct vertical asymptote ***A1***

shape including correct horizontal asymptote ***A1***

$\left(\frac{1}{3},\ 0\right)$ ***A1***

$\left(0,\ -\frac{1}{2}\right)$ ***A1***

Note: Accept $x = \frac{1}{3}$ and $y = -\frac{1}{2}$ marked on the axes.

[4 marks]