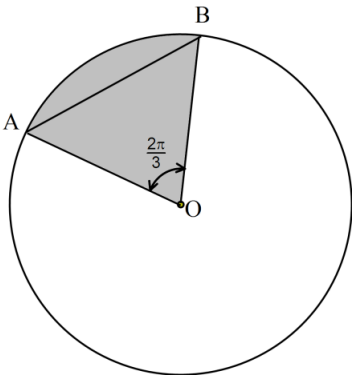
Test - Trigonometric Functions & Equations SOLUTION KEY

■ **Part 1** – NO calculator – Questions 1-6 total marks on test: **45**

1. The diagram shows a circle with centre O. Angle AOB has a measure of $\frac{2\pi}{3}$ radians. The shaded region (a sector of the circle) has an area of 12π cm^2. Find the **exact** area of triangle AOB. **[6 marks]**



$A = \frac{1}{2}\theta r^2$

$12\pi = \frac{1}{2}\left(\frac{2\pi}{3}\right)r^2 \rightarrow 12\pi = \frac{\pi}{3}r^2$

$r^2 = 36 \rightarrow r = 6$

Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(6)(6)\sin\frac{2\pi}{3}$

$= 18\left(\frac{\sqrt{3}}{2}\right)$

area $= 9\sqrt{3}$ cm^2

2. (a) Write the expression $\cos 2x + \sin x$ in terms of $\sin x$ only. **[1 mark]**

(b) Solve the equation $\cos 2x + \sin x = 0$ for $0 \le x \le 2\pi$, giving your answer **exactly**. **[3 marks]**

(a) $\cos 2x + \sin x = 1 - 2\sin^2 x + \sin x$

(b) $\cos 2x + \sin x = 0 \Rightarrow 1 - 2\sin^2 x + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\qquad$ $x = \frac{\pi}{2}$

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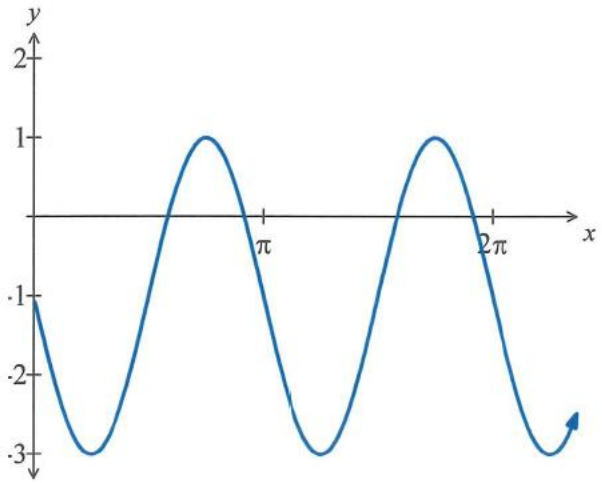
3. The graph of a sine function $y = a\sin(bx) + c$, where a, b and c are integers, is graphed below. Write down the values of a, b and c. **[3 marks]**

$a =$ -2

$b =$ 2

$c =$ -1

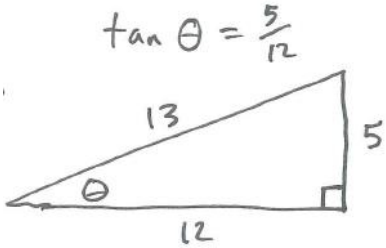


4. Write down the domain, range and period of the function $g(x) = 3\cos(2x)$. **[3 marks]**

domain: $x \in \mathbb{R}$ range: $-3 \le y \le 3$ period: π

5. Given that $\tan\theta = \frac{5}{12}$ and that $0 \le \theta \le \frac{\pi}{2}$, find the exact values of: **[5 marks]**

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\sin 2\theta$



$5^2 + 12^2 = 13^2$

Pythagorean triple

from diagram:

(a) $\sin\theta = \frac{5}{13}$

(b) $\cos\theta = \frac{12}{13}$

double-angle identity

(c) $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \frac{120}{169}$

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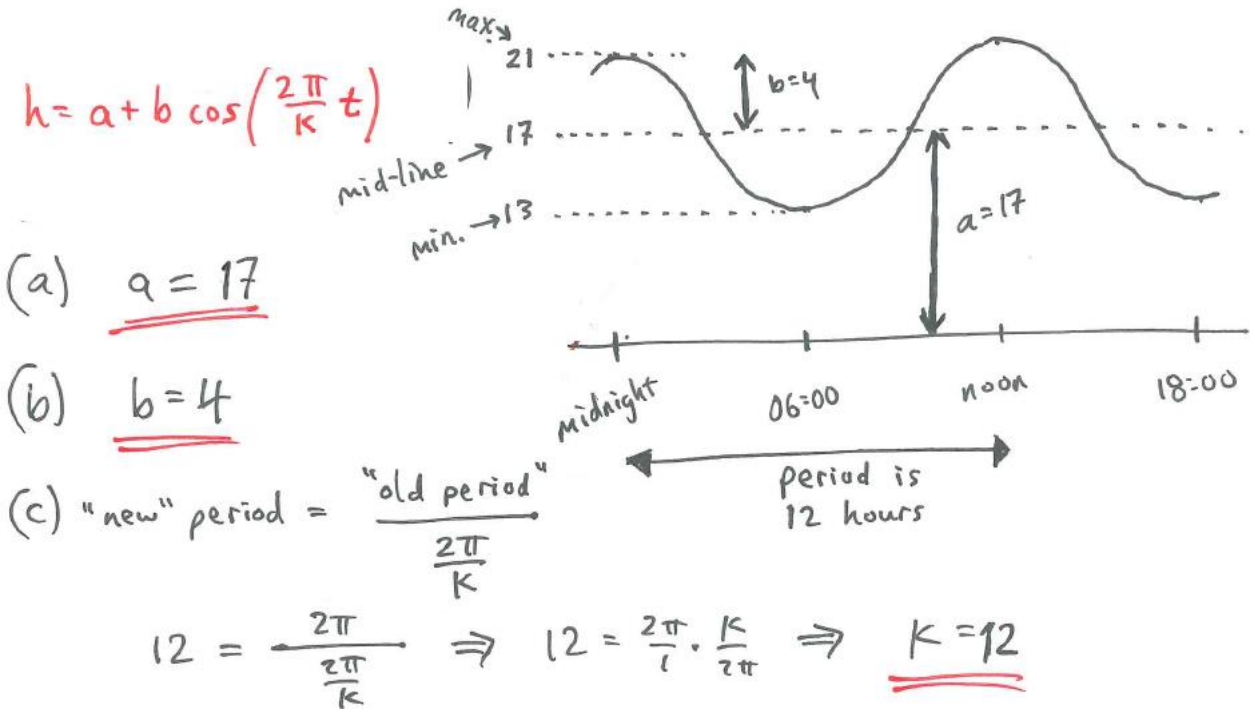
6. The depth of water, h meters, measured at a sea pier t hours after midnight is given by the function $h = a + b\cos\left(\frac{2\pi}{k}t\right)$, where a, b and k are constants.

The water is at a maximum depth of 21 meters at midnight and noon, and is at a minimum depth of 13 m at 06:00 and at 18:00.

Write down the values of: (a) a (b) b (c) k **[6 marks]**



$h = a + b\cos\left(\frac{2\pi}{k}t\right)$

(a) $a = 17$

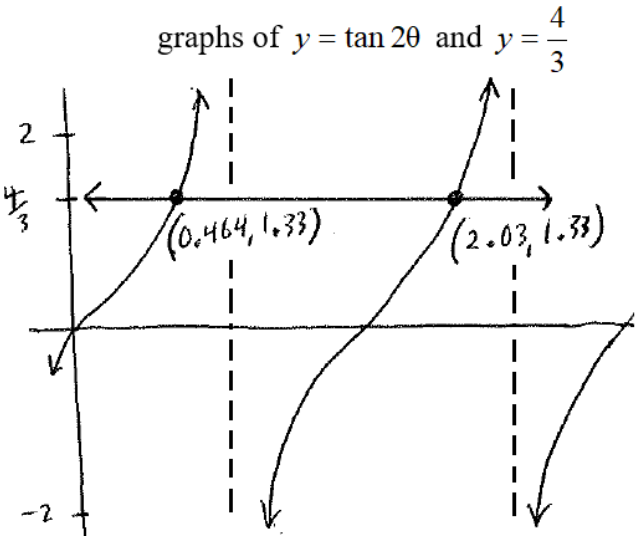
(b) $b = 4$

(c) "new" period = $\frac{\text{"old period"}}{\frac{2\pi}{k}}$

$12 = \frac{2\pi}{\frac{2\pi}{k}} \Rightarrow 12 = \frac{2\pi}{1}\cdot\frac{k}{2\pi} \Rightarrow k = 12$

■ **Part 2** – Calculator allowed – Questions 7-9

7. Find all of the values of θ in the interval $0 \le \theta \le \pi$ that satisfy the equation $\tan 2\theta = \frac{4}{3}$. **[4 marks]**



Solutions are

$\theta \approx 0.464$ or $\theta \approx 2.03$

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8. Solve for θ in the indicated interval. Give any solutions exactly. $4\sin^2\theta = 3, \quad 0 \le \theta < 360°$

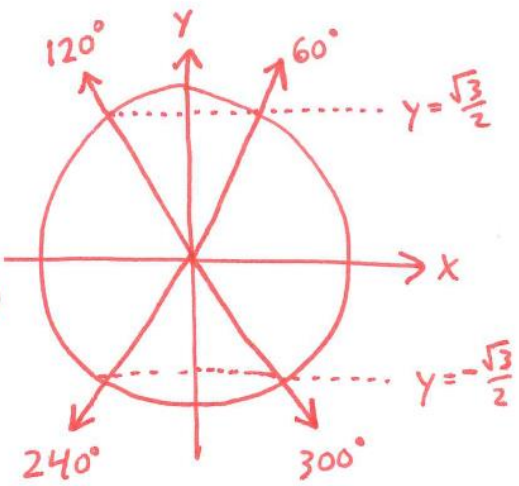
[6 marks]

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$ OR $\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = 60°, 120°$ $\theta = 240°, 300°$



9. The temperature ($C°$) over a 24 hour day in Paris is modelled by the trigonometric function $C = 17 - 6\cos\left(\frac{\pi}{12}t\right)$ where C is degrees Celsius and t is time in hours after midnight.

(a) What is the approximate temperature (to nearest tenth of a degree) at 10:00 am? 22.2°

(b) What is the minimum temperature and when does it occur? 11° at $t = 0$ and $t = 24$

(c) What is the maximum temperature and when does it occur? 23° at $t = 12$

(e) At 18:00 (6 pm), the temperature given by the trigonometric function is $17\,C°$. What other time during the day does the trigonometric function give the same temperature? at 6 am

[8 marks]