# HL/ Polynomials 7.5.2020 *[66 marks]*

1. Let $P(z) = az^3 - 37z^2 + 66z - 10$, where $z \in \mathbb{C}$ and $a \in \mathbb{Z}$. *[6 marks]*

   One of the roots of $P(z) = 0$ is $3 + \mathrm{i}$. Find the value of $a$.

2. The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of $(x-1)$, $(x-2)$ and $(x-3)$. *[5 marks]*

   Find the values of $p$, $q$ and $r$.

Consider the polynomial $q(x) = 3x^3 - 11x^2 + kx + 8$.

3a. Given that $q(x)$ has a factor $(x-4)$, find the value of $k$. *[3 marks]*

3b. Hence or otherwise, factorize $q(x)$ as a product of linear factors. *[3 marks]*

Two distinct roots for the equation $z^4 - 10z^3 + az^2 + bz + 50 = 0$ are $c + \mathrm{i}$ and $2 + \mathrm{i}d$ where $a, b, c, d \in \mathbb{R}, d > 0$.

4a. Write down the other two roots in terms of $c$ and $d$. *[1 mark]*

4b. Find the value of $c$ and the value of $d$. *[6 marks]*

5a. Given that $(x + \mathrm{i}y)^2 = -5 + 12\mathrm{i},\ x,\ y \in \mathbb{R}$ . Show that *[2 marks]*

(i) $x^2 - y^2 = -5$ ;

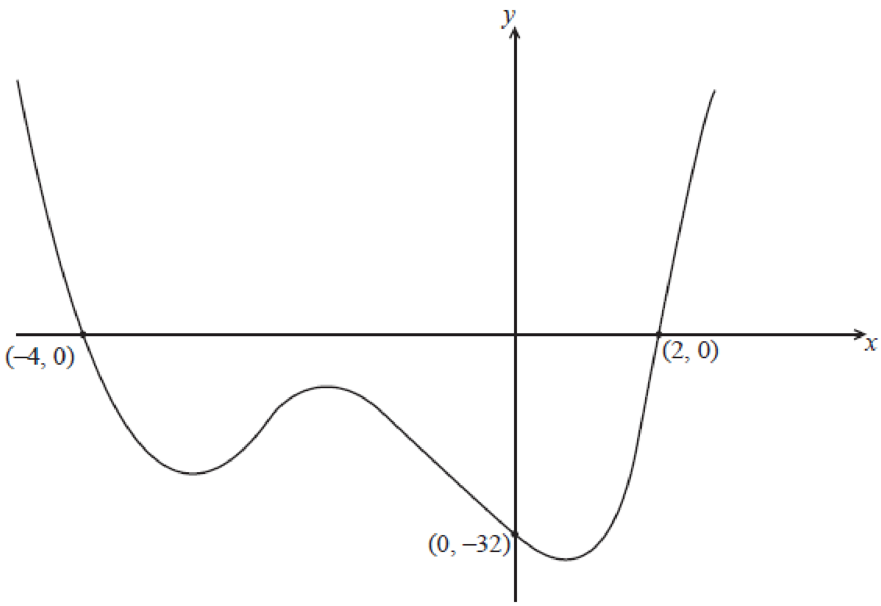
(ii) $xy = 6$ .

5b. Hence find the two square roots of $-5 + 12\mathrm{i}$ . *[5 marks]*

5c. For any complex number *z* , show that $(z^*)^2 = (z^2)^*$ . *[3 marks]*

5d. Hence write down the two square roots of $-5-12\mathrm{i}$ . *[2 marks]*

The graph of a polynomial function *f* of degree 4 is shown below.




5e. Explain why, of the four roots of the equation $f(x)=0$ , two are real and two are complex. *[2 marks]*

5f. The curve passes through the point $(-1,\ -18)$ . Find $f(x)$ in the form *[5 marks]*

$f(x)=(x-a)(x-b)(x^2+cx+d),\ \text{where}\ a,\ b,\ c,\ d\in\mathbb{Z}$ .

5g. Find the two complex roots of the equation $f(x)=0$ in Cartesian form. *[2 marks]*

5h. Draw the four roots on the complex plane (the Argand diagram). *[2 marks]*

5i. Express each of the four roots of the equation in the form $r\mathrm{e}^{\mathrm{i}\theta}$ . *[6 marks]*

6. (a) Show that the complex number i is a root of the equation *[6 marks]*

$$x^4-5x^3+7x^2-5x+6=0\ .$$

(b) Find the other roots of this equation.

Consider the equation $z^3 + az^2 + bz + c = 0$ , where $a$ , $b$, $c \in \mathbb{R}$ . The points in the Argand diagram representing the three roots of the equation form the vertices of a triangle whose area is $9$. Given that one root is $-1 + 3\mathrm{i}$ , find

7. (a) the other two roots; *[7 marks]*

(b) $a$ , $b$ and $c$ .