Composite and inverse functions [42 marks]

1. [Maximum mark: 5] SPM.1.SL.TZ0.5

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

2. [Maximum mark: 6] EXN.1.SL.TZ0.5

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$.

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b. [6]

3. [Maximum mark: 7] 22N.1.SL.TZ0.1

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

(a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. [2]

(b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1,\ 2)$. Find an expression for $h(x)$. [3]

(c) Find $(g \circ h)(0)$. [2]

4. [Maximum mark: 5] 22M.1.SL.TZ2.1

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

(a) Find $g(0)$. [1]

(b) Find $(f \circ g)(0)$. [2]

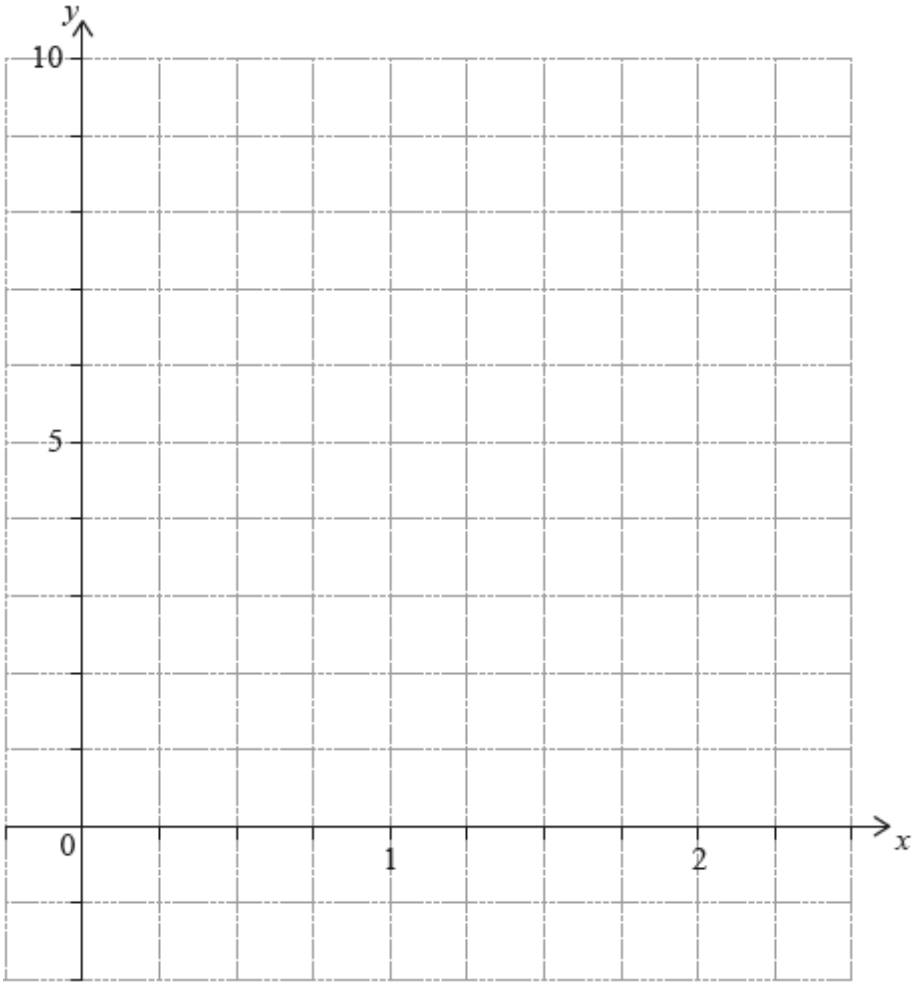
(c) Find the value of x such that $f(x) = 0$. [2]

5. [Maximum mark: 8] 17M.2.SL.TZ2.S_6

Let $f(x) = x^2 - 1$ and $g(x) = x^2 - 2$, for $x \in \mathbb{R}$.

(a) Show that $(f \circ g)(x) = x^4 - 4x^2 + 3$. [2]

(b) On the following grid, sketch the graph of $(f \circ g)(x)$, for $0 \leqslant x \leqslant 2.25$.

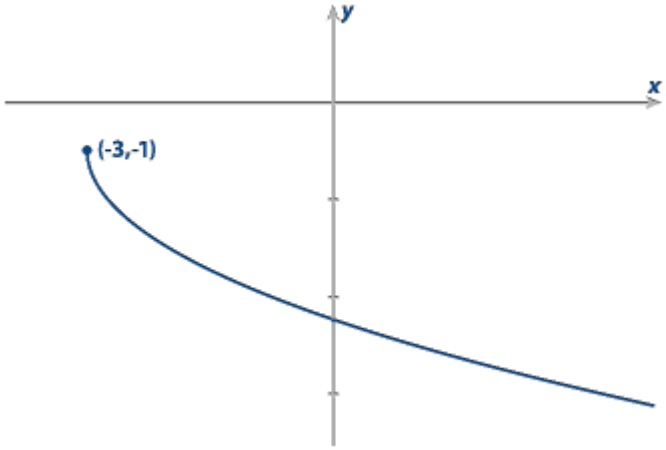


[3]

(c) The equation $(f \circ g)(x) = k$ has exactly two solutions, for $0 \leqslant x \leqslant 2.25$. Find the possible values of k. [3]

6. [Maximum mark: 11] EXN.1.SL.TZ0.8

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

(a) [N/A] [0]

(b) State the range of f. [1]

(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

(d) Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. [5]